

Sketchpad and Shadows

A Technology Supplement to the Shadows Unit

This is a preliminary draft written by Bill Finzer of Key Curriculum Press to demonstrate how *The Geometer's Sketchpad* can be integrated with the teaching of the Integrated Mathematics Program's Shadows unit in the first year of that program.

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Overview

These materials supplement the Interactive Mathematics Program's "Shadows" unit by providing activities based on utilizing The Geometer's Sketchpad. Those classrooms that have access to Macintosh or Windows-capable computers can make use of Sketchpad to strengthen and deepen understanding of the geometric and trigonometric concepts in the unit.

Why Bother? Here are a few of the benefits we believe you and your students will reap if you go to the trouble to use Sketchpad in this unit:

- The process of constructing a mathematical model of the shadow experiment leads to a greater understanding of the geometric constraints underlying the physical situation. Manipulating the completed model provides a way, in addition to the physical experiment, to gather data and check out conjectures.
- Once having learned to use Sketchpad in this unit, you and your students can use it to good advantage in other IMP units.
- The fundamental concepts in the unit such as similarity and rigidity can become quite vivid and clear when embodied in a dynamically draggable sketch.

Does this mean I don't have to gather the flashlights for the experiment? Absolutely not. Using Sketchpad comes after the physical experiment and is used as a tool to model the physical situation.

Will the unit take longer if I use Sketchpad? Just a little. You need to insert some days near the beginning, but you gain most of them back near the end.

Do I need to be familiar with Sketchpad to use this supplement? Yes. If you have not used Sketchpad, you will need to work through the first four tutorials that come with the software, and you should work through the

investigations in this supplement before you start teaching the unit.

Do my students need to be familiar with Sketchpad before we can use this supplement? We don't think so. We've made the first supplementary day's activity a kind of tutorial for Sketchpad as well as an investigation of concepts relevant to the unit.

What equipment do I need? How can my students and I best use it? This supplement assumes that there is one computer for every two students. Given this, a group of four can work as two pairs on two machines. (Four students using one machine provides too little access to the screen and mouse.)

Sketchpad projected on an overhead projector using an LCD panel serves as a fabulous electronic blackboard for this unit. Students can make presentations of their discoveries and you can show some convincing visual demonstrations of some of the properties of similar figures.

You do not need a printer, but if you have one, students will make good use of it by printing their annotated sketches.

Planning

Here is a plan for how to restructure the Shadows unit to take advantage of Sketchpad. Three days are added to the beginning part of the unit and two are gained back near the end. The days marked with an asterisk could be done without any of the other days.

**Day 4a: Exploring Right Triangles.* Students get a guided tour through some Sketchpad construction techniques that they will need in later activities.

*Day 4b: *Modeling Shadows*. Students construct a sketch that models the physical experiment.

*Day 4c: *Shadows on the Screen*. Students supplement the data they gathered from the physical experiment with data from their Sketchpad model.

Day 7: *Similarity and Counterexamples*. Students use Sketchpad to construct counterexamples.

Day 8: *Why are Triangles Special?* This day's investigation can be done to good advantage with Sketchpad.

Day 9: *Triangular Possibilities*. Do it with Sketchpad. It's quite possible that you can save a day in days 7-9 because much work can proceed faster with Sketchpad than without.

Day 13-14: *Total Turns*, renamed *Outside Angles*. Sketchpad makes an ideal data gathering tool for this investigation. Several visually convincing demonstrations are possible here as well.

*Day 17: *A Shadow of a Doubt*. Students use the Sketchpad model they constructed on day 4c to help with the formulation of the algebraic relationship between L , H , and D . They should reach the long term goal of the unit on this day instead of on day 19.

POW 18: *Billiard Bankshots*. If your students have access to Sketchpad outside of class time, they can use Sketchpad to model this problem and to help solve it.

*Day 19: *Finding a Function*. This day is not needed. Its work got accomplished on day 17.

*Day 20: *The Return of the Tree*. Students do this investigation with Sketchpad. It doesn't take long. The material from day 21 on \sin , \cos , and \tan gets integrated into this day

**Day 21: Sin, Cos, and Tan Revealed.* This day has been combined with day 20.

Using this Supplement

For the three new days, here labeled 4a, 4b, and 4c, you'll find a complete lesson plan including homework. We've tried to adhere to the form of teacher notes found in the standard IMP unit.

For the remainder of the days in the unit, we've included supplementary notes for those days for which use of Sketchpad seems desirable. *You will still need to read the standard IMP unit notes for those days* as we haven't duplicated the standard notes in this supplement.

Student Materials (Teacher Edition)

Day 4a: Exploring Right Triangles

Mathematical Topics

- Making conjectures about the properties of pairs of triangles that have parallel bases.

Preparation

You will need the following handouts:

- Exploring Right Triangles
- Homework 4a: Euclid's Tools

Students will work with Sketchpad and so will need access to computers, one computer for every two students.

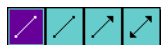
Demonstrating Sketchpad

If you have access to a demonstration screen such as an LCD panel mounted on an overhead screen, you can, if you choose, demonstrate The Geometer's Sketchpad to students before they attempt the guided tour on the handout. Different teachers have different styles with respect to how much they like to show students beforehand about how a new piece of software works. You can, if you like, have the students simply plunge into the investigation without any demonstration.

Here we outline a demonstration that you can do in about ten minutes.



The tools: Explain that the four tools on the left of the sketch window are for drawing and dragging objects.



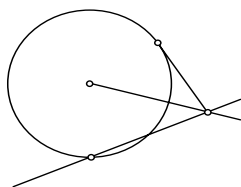
Start with the segment tool. Draw some segments. Show that when you hold the mouse button down on the tool icon, get a choice of

drawing segments, rays, or lines. Draw a few of each of these.

Show how the arrow tool can be used to drag something once you have drawn it. Use the arrow tool frequently throughout the demonstration and emphasize that one should drag to see whether you have made what you really wanted and to investigate the properties of what you have made.

Draw some circles. Don't spend much time with them because students will not be using the circle tool in their work in this unit.

Draw some points. Show that if you place a point on a line or circle, it is constrained to remain there.



More constraints: Make a figure (anything will do) in which segments, rays, lines, and circles share some control points. (Control points are the points that define the geometric object such as the center point of a circle or the end points of a segment.) Show that as you drag things around with the arrow tool (anything, not just the points), the drawing holds together.

Construct	Transform	Mea
Point On Object		
Point At Intersection	⌘I	
Point At Midpoint	⌘M	
Segment		⌘L
Perpendicular Line		
Parallel Line		
Angle Bisector		
Circle By Center•Point		
Circle By Center•Radius		
Polygon Interior		⌘P
Circle Interior		
Construction Help...		

Construction: Point out that there are other important relationships that geometric objects can have. Demonstrate construction of a line through a point parallel to another line. Note that you cannot do any construction until you have chosen the objects on which you wish the construction to take place.

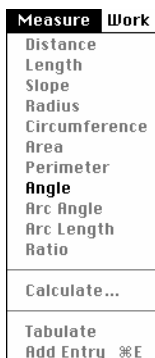
Use the arrow tool to show that no matter how you drag things around, the two lines will remain parallel.

Similarly, construct a line through a point perpendicular to another line. Do this both when the point is on the line and when it is off the line. Again, demonstrate how the two lines

remain perpendicular no matter how you drag things around.



Labels: Show how to label a point or a line. Show how you can change a label that comes up by double-clicking on it.

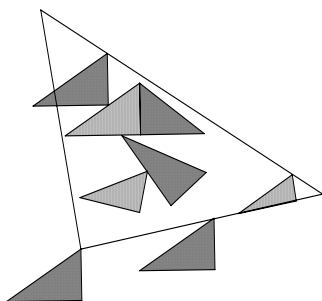


Measurements: Demonstrate how to measure an angle and measure a length. Show how the measurement will update while things in the sketch are dragged.

Exploring Right Triangles

Explain to students that they are about to begin using a new piece of software that is very useful in mathematics generally and useful for working on the shadows question in particular. Emphasize that it will take a few days of using the software before they are comfortable. Today they will go through a guided tutorial that will show them step-by-step what to do. Working in pairs, they should be able to figure most things out. If they can't, they can ask for help from the other pair in their group. Asking the teacher should be a last resort.

Exploring Right Triangles



Work in pairs on this tutorial to learn how to use The Geometer's Sketchpad as a tool that will help you investigate the shadows problem. One person can read the tutorial while the other carries out the instructions on the computer. Trade roles several times so both of you get a chance to try using the computer.

<Say something here about what they should prepare for discussion with the rest of the class.>

Any Old Triangle



Click once on the segment tool. When you move the mouse pointer into the drawing area of the window, notice that it becomes a crosshair. You draw a segment by holding down the mouse button in one spot and dragging to another spot before releasing.



Use the segment tool to draw a triangle.



Click once on the arrow tool. Move the mouse pointer over one of the vertices, hold the button down, and drag the vertex. Try dragging the other two vertices. Finally, try dragging the sides of the triangle.

Label each of the statements below true or false.

- ___ 1. When you drag a vertex of the triangle, the side opposite that vertex does not change length.
- ___ 2. When you drag a side of the triangle, all three sides change length.
- ___ 3. You can drag the triangle so that it is a right triangle.

How to Label Points and Segments



Click once on the label tool. Notice how the mouse pointer becomes a hand when you move it into the drawing area. When you move the finger on top of something you can label—a point, line, or circle—it highlights. When the hand is highlighted, click the mouse button. The object you clicked on will get a label.



To make the label go away, click on the object with the label tool again. One click to show the label, another click to hide it.



Point the finger at a label. Notice that the hand gets a little letter "A" in it. Hold down the mouse button and you can move the label around. Notice, though, that you can't move the label very far from its object.



Double-click on a label with the hand's finger. You will get a dialog box with which you can change the label to whatever you want. Click the OK button when you have what you want.

Experiment with labels in order to decide which of the following statements are true and which are false:

- ___ 4. Line segments can have labels.
- ___ 5. When a point moves, its label moves with it.
- ___ 6. Once you have changed a label, you can't hide that label.

How to Undo What You Did



The Edit menu contains the Undo command. Each time you select it, the last thing you did gets undone. You can keep "undoing" all the way back to the beginning of your work. You can also use the Redo command to redo something you just undid. Got it?

The absolute best way to fix most mistakes in Sketchpad is to keep undoing until the mistake is gone.

True or false?

- ___ 7. When you drag a point, the Undo command does *not* put the point back where it was.

How to Construct a Right Triangle

The triangle you made could be *any* triangle because you could drag the vertices anywhere you wanted. Now you'll learn how to construct a triangle that will always stay a *right* triangle, a triangle with a 90° angle in it.

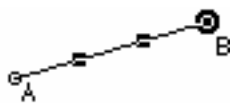


Choose New Sketch from the File menu. This will give you a blank sketch in which you can begin a new construction.

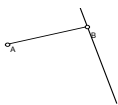


Using the segment tool, draw a segment. Label its endpoints A and B.

Next you'll construct a line through point B perpendicular to the segment.



Using the arrow tool, select point B . Now hold down the Shift key and click on the segment. You should see that both the point and the segment are selected, as shown at left.

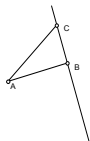


Choose the Perpendicular Line command from the Construct menu. You should get a line through point B perpendicular to segment AB .

Try dragging various points around and notice how the line through point B always remains perpendicular to segment AB .

True or false?

- ___ 8. Dragging point A does not change the orientation (slope) of the perpendicular line through B .
- ___ 9. Dragging the perpendicular line drags point A .



Using the segment tool, draw a segment from somewhere on the perpendicular line to point A .

Using the label tool, label the new point C .

Using the arrow tool, select the perpendicular line.

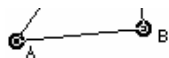
Issue the Hide Line command from the Display menu. This should make the perpendicular line disappear.

Using the segment tool, connect points B and C to complete your right triangle.

True or false?

- ___ 10. Point C can only be dragged along the hidden perpendicular line.
- ___ 11. Dragging point A changes the length of segment BC .

How to Measure Distances and Lengths



Using the arrow tool select point A (without holding down the Shift key). Then hold down the Shift key and select point B . Both points should highlight as shown at left.



If the two points are properly selected, you should be able to issue the Distance command from the Measure menu.

Distance(A to B) = 1.00 inches

The command's result is to place the desired measurement in the sketch as shown at left.

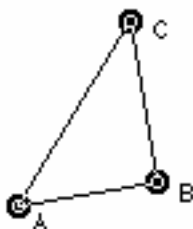
Length(Segment m) = 1.00 inches

You can also measure the length of a segment. To do so, select a segment with the arrow tool. Then choose the Length command from the Measure menu.

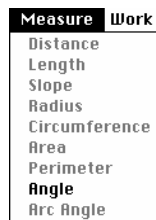
Decide about the truth or falsity of each of the following statements by taking some measurements of parts of your right triangle.

- ___ 12. No matter how you drag the points, the distance between points A and C stays greater than the distance between A and B .
- ___ 13. The length of segment BC can never be made zero.

How to Measure Angles



An angle measurement requires that you select three points, the middle of which is the vertex of the angle whose measure you want. To measure the angle at vertex B , select point A with the arrow tool, then hold down the Shift key and select points B and C in that order.



With the three points properly selected, you should be able to issue the Angle command from the Measure menu, resulting in the measurement appearing in the sketch window.

Measure the angles at the other two vertices of your right triangle.

By dragging parts of your triangle around and observing your measurements, answer each of the following questions.

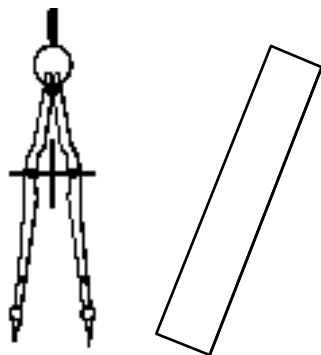
14. What is the sum of the measure of angle A and the measure of angle B ? Is this sum always the same?
15. What is the measure of angle A when the length of AB equals the length of BC ?
16. When the length of AC is 2 inches and the length of BC is 1 inch, what is the measure of angle A ?

If you finish early, come up with some statements that seem to be true about your right triangle. Be prepared to share these with the rest of the class.

Homework 4a: Euclid's Tools

The drawing tools that Euclid based his geometry on were the straightedge and compass. Sketchpad offers these tools in the toolbar on the left edge of a window. This homework involves students in doing some simple constructions with the physical drawing tools.

Demonstrate one of the first two constructions. Be careful to point out the difference between using the straightedge as a straightedge and using it as a ruler. You might do this by showing how you could solve construction three using a ruler by measuring the distance between the two points and then measuring half that distance from one point to find the midpoint. Explain that this kind of construction is not allowed.

Homework 4a: Euclid's Tools

You need to have a sharp pencil, a compass and a straightedge to do this homework.

For each item below decide whether you can use your three tools to accomplish the construction. (Note that you are not allowed to use the straightedge as a ruler!) If you can do it, show the result and explain in words how you did it. If you cannot do it, explain why the methods you tried did not work.

1. Given two points, construct a line passing through both of them.
2. Given two points, construct a circle whose center is at one point and which passes through the second point.
3. Given two points, construct a point exactly halfway between them.
4. Construct a right angle.
5. Construct two parallel lines. (Using the opposite parallel edges of the straightedge is *not* a valid construction!)

Day 4b: Modeling Shadows

Mathematical Topics

- Constructing a mathematical model of a physical situation

Preparation

You will need the following handouts:

- Modeling Shadows
- Homework 4b: Modeling Madness

Students will work with Sketchpad and so will need access to computers, one computer for every two students.

Discussion of Homework 4a: Euclid's Tools

Let groups talk over their homework, being prepared to have anyone from their group demonstrate any of the constructions (or explain why it couldn't be done).

Choose a group and student at random to demonstrate each of the constructions.

If you like, you can follow along with the student using Sketchpad as a presentation tool. Be sure, however, not to use the Construction shortcuts for construction. Stick to the circle and line tools.

Modeling Shadows

Explain that the day's activity involves constructing a computer model of the shadow experiment using The Geometer's Sketchpad. Stress that for the sketch they make to be useful, it should be easy to drag things around in the sketch and still have the model accurate.

As on the previous day, students should work in pairs at computers, but pairs should be grouped so

that groups of four are available for review of what they have done.

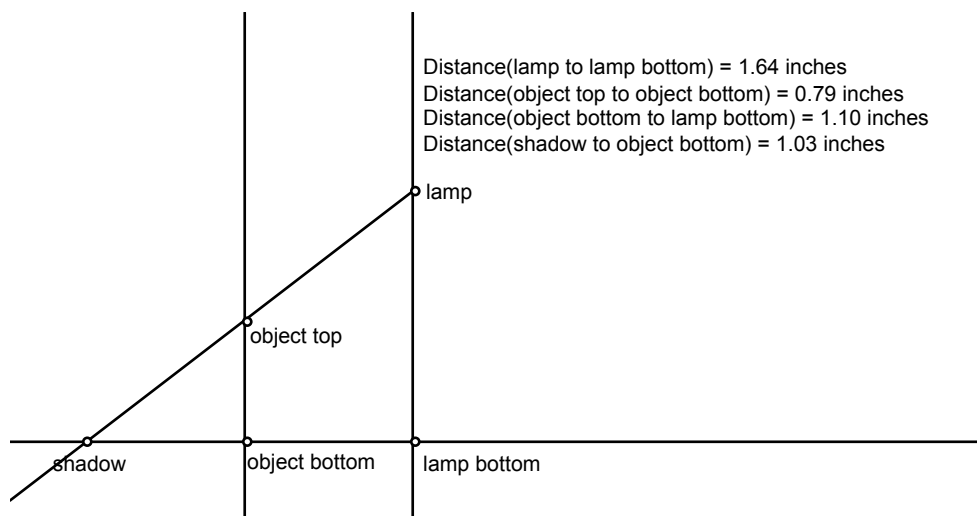
If students are having trouble getting reasonable models, stop part way through the period and have them discuss what they have been doing. Attempt to clear up problems they are having.

Just before the period ends, ask volunteers to demonstrate sketches that they created. Some students may have had trouble creating a table of measurements, so demonstrate how that works.

Make sure groups save their models on disk! This could be on floppy disks, or it could be on the computer's hard disk. They will be using them during the following class period and again on day 17.

Modeling Shadows

Your goal in this activity is to construct a model of the shadows experiment using The Geometer's Sketchpad. You will use the model to gather data that will supplement the data you gather from the experiment.



The figure above shows what you want. But beware, good looks are not sufficient. The parts of your model must *behave* well, too.



Hint: You will almost certainly need to use the ray tool.

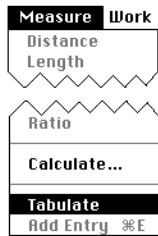
Another hint: If your model doesn't behave the way you want it to, don't be afraid to start over again with a new sketch.

Here is checklist of questions to use to verify that your model behaves well. *Write down an answer to each question.*

1. What distances change when you drag the lamp?
2. What distances change when you drag the lamp bottom?
3. What distances change when you drag the object top?
4. What distances change when you drag the object bottom?

How to Make a Table of Measurements

Collecting measurements in a table will save you from having to copy numbers onto paper.



Using the arrow tool, select one of the measurements. Hold down the shift key and select the other measurements.

Choose Tabulate from the Measure menu. You should get a table similar to the one below.

Distance(lamp to lamp bottom)	1.64
Distance(object top to object...	0.79
Distance(object bottom to lam...	1.10
Distance(shadow to object bot...	1.03



Choose the label tool. With the tip of the tool's finger, double-click on a row label. Type in a new row label that will be easier to understand.



When you are finished changing the row labels, your table should look similar to the one at left.

Using the arrow tool, select the main part of the table, the part where the numbers are.



Choose the Flip Direction command from the Measure menu. This should orient your table with the labels at the top of columns as shown at left.

Change something, such as the height of the object, in your sketch. Now double-click on the main part of the table. Notice that a new row of entries gets added to your table.

If You Have Time

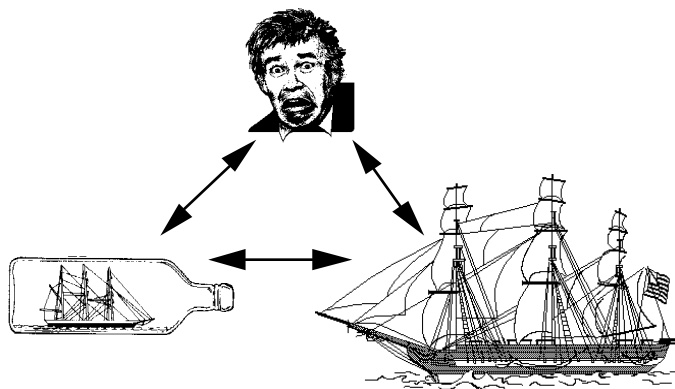
Experiment with your model of the shadows experiment. Write down some observations about how the model works.

Important! Be sure to save your sketch on disk. You will need to use it on the next day to gather data.

Homework 4b: Modeling Madness

The point of this homework is to tie the day's experience with more commonplace models

Homework 4b: Modeling Madness



Each thing listed below can be considered a model of something. For each, answer the following questions:

- What is it a model of?
- Under what circumstances might you prefer using the model to using the real thing? And why?
- List two questions you would be able to answer using the model.

1. A road map of the United States.
2. A mockup of an airplane wing placed in a wind tunnel.
3. An architect's plans for a skyscraper.
4. A flight simulator.
5. A tailor's pattern for a jacket.
6. A rug diagram for the game of Pig.

Day 4c: Shadows on the Screen

Mathematical Topics

- Gathering and organizing data
- Controlling variables
- Verifying a theory

Preparation

You will need the following handouts:

- Modeling Shadows
- Homework 4c: *Getting One from Another*

Students will work with Sketchpad and so will need access to computers, one computer for every two students.

Discussion of Homework 4b: Modeling Madness

Focus discussion on the questions that the models can help you answer. For each of the five models, have students volunteer questions and write them up on the overhead. Ask, "Which of these models would require use of mathematics to use, and why do you think so?"

Shadows on the Screen

On day 4, before the work with Sketchpad began, groups of students had generated some kinds of formulas for the various possible relationships among the shadow experiment variables. Now they are in a position to gather more data from their Sketchpad model. Purposes they may put this data to include verifying that the Sketchpad model data follow the same pattern as does the physical data, and attempting to find a formula for a difficult relationship now that there is more data.

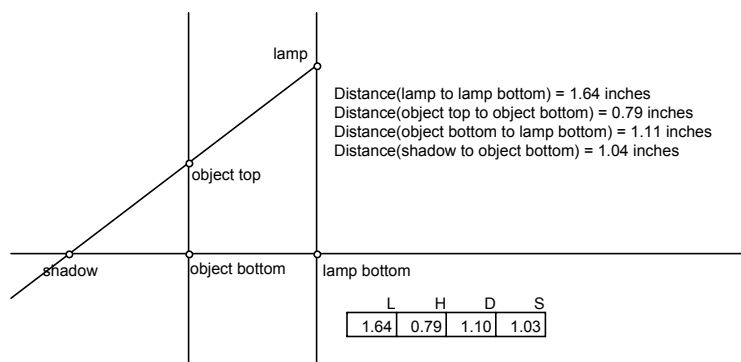
Field testers: What do they come up with? Does it relate to Day 4 formulas?

If students have access to a printer, they will find it helpful to print their sketch after they have gathered data as it is easier to refer to a print-out while graphing than it is to refer to the computer screen.

Ask for reports of progress made in fitting curves to data. Students may notice that the data from their Sketchpad model has much less “noise” in it than their data from the physical experiment.

Shadows on the Screen

Open the sketch you saved at the end of the last class period. Presumably it looks similar to the one shown below.



Your job here is to perform an “experiment” using your Sketchpad model of the shadows situation. You will gather and analyze data for the length of the shadow as one of the other three variables changes.

1. Choose one of the variables, D , H , or L , to change while holding the other two constant. Take at least ten measurements. (To add an entry to a table, double-click the numbers in the table.)
2. On graph paper make a plot of your data. If, for example, you chose to vary D , then put D on the horizontal axis and S on the vertical axis.
 - a) In what ways does it look *similar* to the graphs produced from the physical data?
 - b) In what ways does your graph look *different* from the graphs produced from the physical data?

3. Attempt to fit an equation through your data points. You can use any of the three methods discussed on the *Looking for Equations* worksheet.
4. Repeat steps 1–3 with a different variable.

Homework 4c: Getting One from Another

Introduce the homework by asking students what are some examples of things that you could do to one quadrilateral to get another. Some ideas they might have include shrinking or growing it, keeping all the sides the same length, adding something to each side, and keeping the angles the same.

Going Over Homework 4c on the Next Day

Included on the disk that comes with this supplement is a sketch of the homework problem. You can use it to try out students ideas of how each of the numbered quadrilaterals was obtained from $ABCD$. You can do this partly by changing $ABCD$ and watching how the others change, and by measuring side lengths and angles.

Each quadrilateral was produced from $ABCD$ as follows:

1. $ABCD$ was shrunk by half. You can determine this by measuring the angles and seeing the corresponding angles are equal and also measuring the ratio of side lengths to find that this ratio is $1/2$ for all sides.
2. Each side of the new quadrilateral has half the corresponding side of $ABCD$. But the corresponding angles are not equal.
3. $ABCD$ was dilated by half. You can determine this by measuring the angles and seeing the corresponding angles are equal and also measuring the ratio of side lengths to find that this ratio is $2/1$ for all sides.
4. This quadrilateral is identical to $ABCD$.
5. The angles on this quadrilateral have the same measure as the corresponding angles on $ABCD$. But the new quadrilateral is not similar

to the original because the sides are not in a constant ratio.

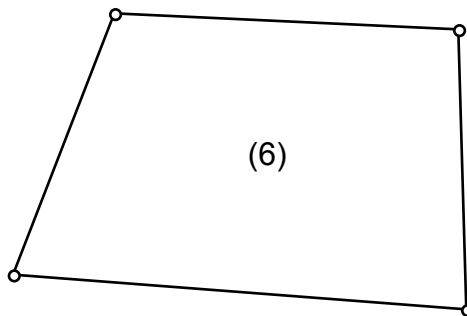
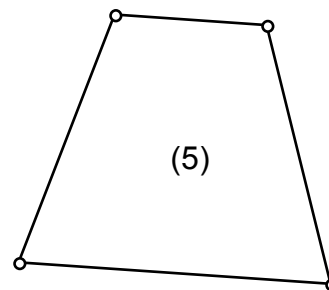
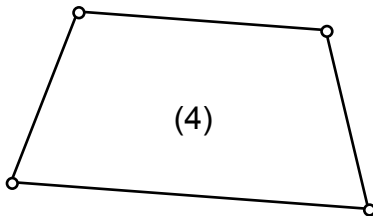
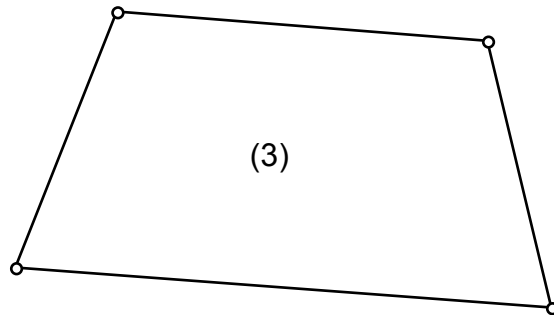
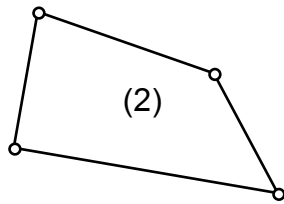
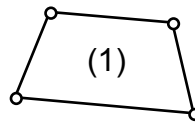
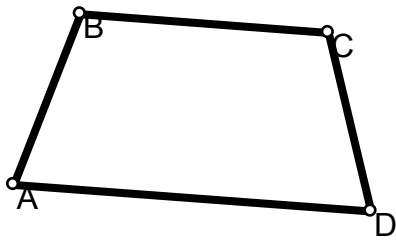
6. Each side of the new quadrilateral is 0.5 inches longer than the corresponding side of $ABCD$.

Homework 4c: Getting One from Another

Each of the seven numbered quadrilaterals below was produced by doing something to quadrilateral $ABCD$ in the upper left corner. For each,

- a) describe how it was produced from $ABCD$, and
- b) describe how you decided this.

(You'll need a ruler and protractor.)



Day 7: Similarity and Counterexamples

This day's activity can be done with Sketchpad. Using Sketchpad will encourage students to go farther in finding counterexamples because it is easy to construct new figures.

Preparation

You will need the following handout – modified from the one in the standard unit:

- Is There a Counterexample?

You may choose to use the following additional handout and script:

- How to Copy Lengths and Angles – a Tutorial
- Load the “Copy Angles Script” on the computers' disks.

The Investigation

Teachers: Did you elect to use the handout? Please comment.

The optional handout below provides a short tutorial on copying angles and lengths. Students *may* use these techniques for the investigation, but they can also base their investigation on unconstrained drawings.

If you use the tutorial, you will need to provide students with the script “Copy Angles Script.”

How to Copy Lengths and Angles – a Tutorial



It often happens as you construct something with Sketchpad that you would like one length to stay equal to another or one angle to stay the same as another. This tutorial will show you how to do that.

How to Copy a Length



Using the segment tool, draw a segment.



Label the end points *A* and *B*.

Using the point tool, place a point in your sketch.

Label this point *C*.



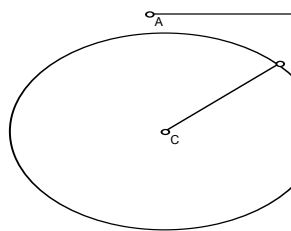
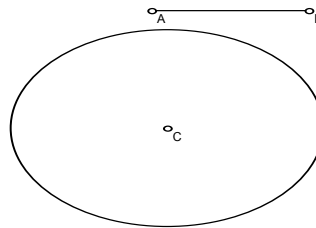
Using the arrow tool, select point *C*.

Holding down the Shift key, select the segment between points *A* and *B*.



Construct	Transform	Mea
Point On Object		
Point At Intersection	⌘I	
Point At Midpoint	⌘M	
Segment		⌘L
Perpendicular Line		
Parallel Line		
Angle Bisector		
Circle By Center+Point		
Circle By Center+Radius		
Polygon Interior		⌘P
Circle Interior		
Construction Help...		

Choose Circle by Center + Radius from the Construct menu. The result should be a circle centered at *C* with radius equal to the length of *AB*.



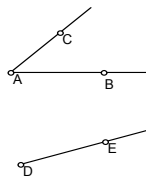
Using the segment tool, draw a segment from point *C* to any place on the circumference of the circle.

Using the arrow tool, select the circle.

Choose Hide Circle from the Display menu.

Drag points *A* and *B*. Notice that the length of the new segment remains equal to the length of *AB*, but its direction is independent.

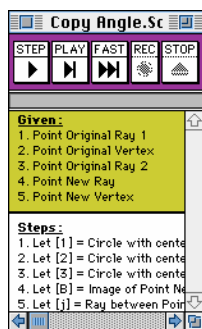
How to Copy an Angle



Use the Ray tool to draw a angle BAC as shown at left.

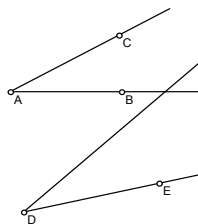
Draw a ray DE as shown below and to the left. The task is to duplicate angle BAC at vertex D .

Choose the Open command from the file menu and open the script entitled CopyAnglesScript.



The script you have opened will automatically copy an angle. Notice that the top part of the script window contains a list of five “givens.” These are the three points that define the original angle plus the two points that define one side of the new angle.

Using the arrow tool, select point B . Holding down the Shift key, click on points A , C , D , and E in that order.



Press the Fast button in the top bar of the script window.

A new ray should appear from point D .

Drag point C . Notice how the angle at D remains equal to angle BAC .

Below is a handout for the main activity of the day – modified from the one in the IMP teacher’s notes.

Is There a Counterexample?



Use Sketchpad to attempt to construct counterexamples to each of the statements below. If you are able to find one counterexample, go on and find a second.

Explain why your examples disprove the given statement.

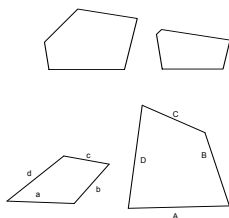
If you do not think a counterexample exists, then explain why you think so.

1. If two polygons have their corresponding angles equal, then the polygons are similar.
2. If two polygons have their corresponding sides proportional, then the polygons are similar.
3. If a triangle has two equal sides, then it will also have two equal angles.

Sharing Counterexamples

Students can demonstrate their sketches on the classroom overhead using Sketchpad, or, if no projection system exists, on a transparency.

It will be easier to begin with items 1 and 2, for which there are counterexamples. Some examples of counterexamples are:



- 1) a square and a non-square rectangle, or the two pentagons shown at left.
- 2) a square and a non-square rhombus, or the two quadrilaterals at left for which the sides of the right hand figure are 1.5 times those of the left figure, but they two are not proportional.

Students may find more exotic counterexamples than these.

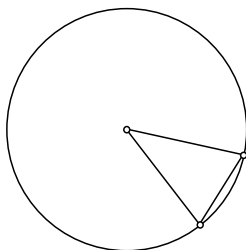
It is important, in preparation for tonight's homework, to ask the groups presenting how they knew they had a counterexample. Make sure they verify both that the polygons fit the hypothesis and that they are not similar.

Specifically go over how to check the proportionality of sides. Ideally this should come from the presenters. However, they may have counterexamples that are obviously dissimilar and see no need to check the proportionality of the side lengths.

In the case of item 3, students will probably believe that there are no counterexamples, but may be unsure about how to articulate this. They should realize that, if there really are no counterexamples, then the statement is true, but that *not finding a*

counterexample is not the same as *none existing*. In other words, the fact that they didn't find a counterexample does not mean that they have proved that the statement is true.

Angle(ACB) = 69.93°
Angle(ABC) = 69.93°



If none of the students have done so, demonstrate construction of an isosceles triangle using two radius segments of a circle. Measure the two angles opposite the equal sides. As you drag one of the vertices around the circle and notice that the two angles remain the same. Again, this demonstration is not a proof that there are no counterexamples. In particular, it doesn't by itself explain *why* the two angles have to be equal.

It's okay to tell them that the statement is actually true. Let them talk about what makes them think so. They may be able to give a good intuitive explanation based on symmetry.

Day 8: Why are Triangles Special?

Sketchpad provides a great medium for playing with triangles and learning what makes them special; that is, rigid. Normally polygons made in Sketchpad are completely elastic, so students will learn how to construct polygons with fixed length sides.

Preparation

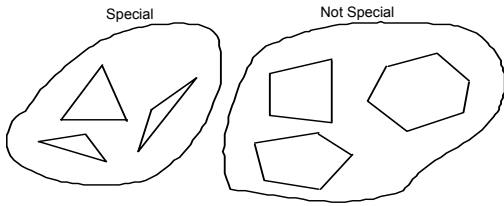
You will need the following additional handout:

- Why Are Triangles Special?

You won't need any special materials.

Why are Triangles Special?

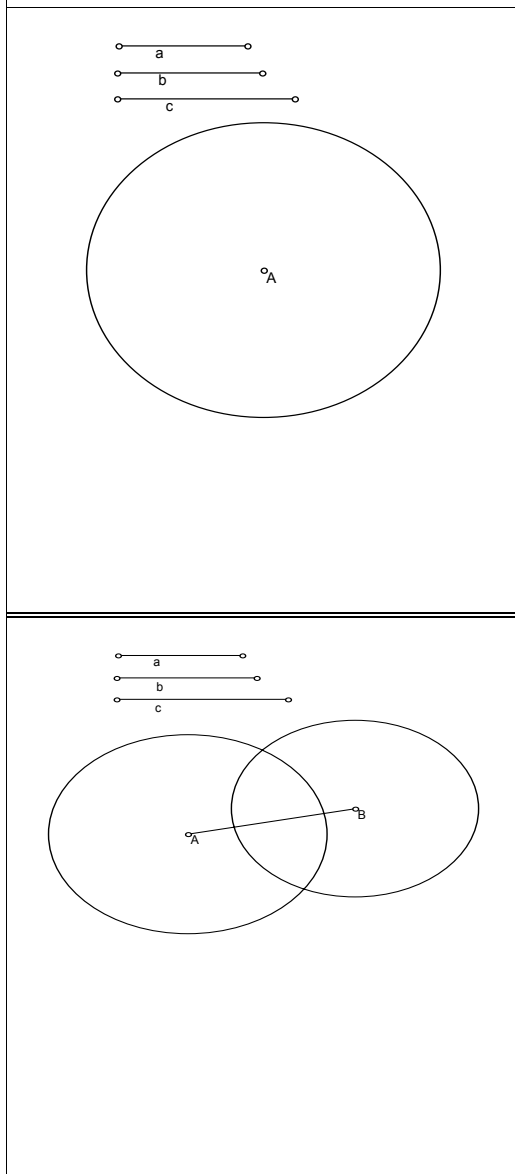
If you have a demonstration computer, you can run through the first investigation, showing the technique for constructing a triangle with fixed length sides.

Why are Triangles Special?	
	Triangles have some properties that separate them from your average polygon. In this investigation you will find out about these properties and relate them to similarity.

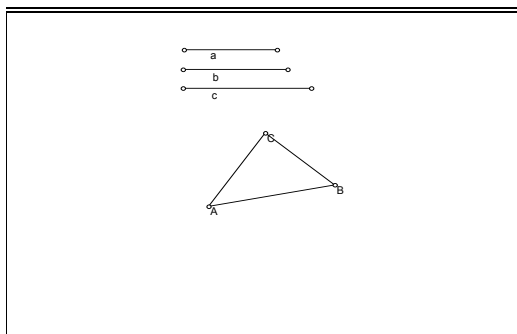
I. Constructing a Triangle with Fixed Length Sides

Normally, a triangle you draw with Sketchpad has sides like rubber bands. Here you learn how to draw one with fixed length sides.

Start with a new sketch.



1. Place three line segments in your sketch. These will be the lengths of the sides of your triangle. Label them a , b , and c .
2. Place a point in your sketch to serve as the first vertex of the triangle. Label it A .
3. Select point A and segment c (using the Shift key).
4. From the Construct menu choose Circle by Center+ Radius.
5. Draw a line from A to some point on the circumference of the circle.
6. Hide the circle.
7. Label the point on the circumference B .
8. Construct a circle with center A and radius b .
9. Construct a circle with center B and radius a .

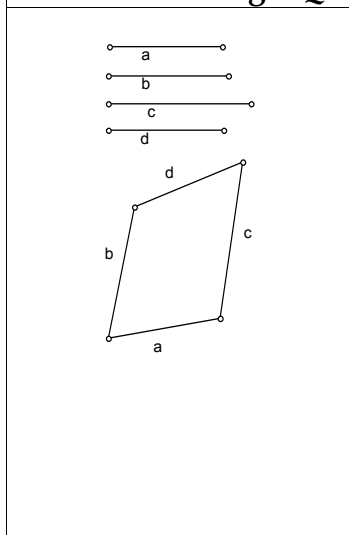


10. Construct a point at the intersection of the two circles.
11. Label the point C.
12. Hide the two circles.
13. Draw segments AC and BC .

Drag the points in your sketch so that you can get a sense of what triangles are possible and what ones are not.

Write five observations based on your sketch. Be prepared to share these with the rest of the class.

II. Constructing a Quadrilateral with Fixed Length Sides



Using the same techniques that you used to construct a triangle with fixed length sides, construct a quadrilateral with fixed length sides. Your sketch will look similar to the one shown at left.

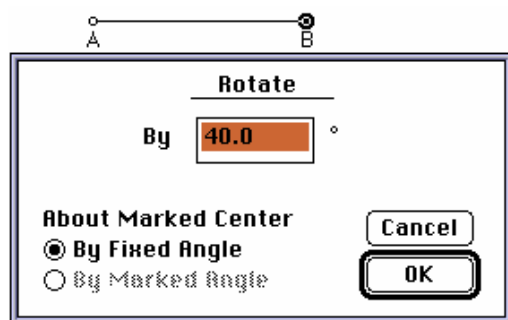
What differences do you observe when you compare your triangle sketch with your quadrilateral sketch?

Suppose two triangles have their corresponding sides proportional. What does that imply about their angles?

Must two triangles with corresponding sides proportional be similar or could they have different angles? How about for quadrilaterals? What do you think would be true for polygons with more sides?

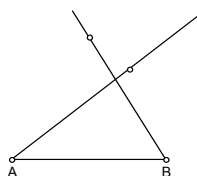
III. Constructing a Triangle with Fixed Angles

In the previous activity you constructed polygons whose side lengths were rigid. Now you'll construct a triangle by fixing the angles and seeing what is left free to change.



1. Draw line segment AB .
2. Select point A and choose the Mark Center "A" command from the Transform menu.
3. Select point B and choose the Rotate command from the Transform menu.

4. In the dialog box that comes up, specify 40 degrees as the rotation angle and press OK.



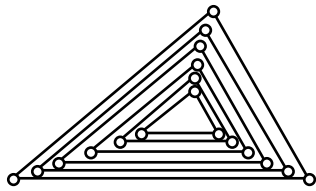
5. Use the ray tool connect point A to the rotated point B .
6. Repeat steps 2–5 using point B as the center of a *negative* 60 degree rotation of point A . Your sketch should look similar to the one at left.

7. Place a point at the intersection of the two rays. Label it C .
8. Hide the two rotated points and the two rays.
9. Draw segments AC and BC .

Experiment with your triangle. What things can you do to it? What things can you *not* do to it?

10. Using the arrow tool select the entire triangle, ABC . (It will be helpful if you hide the point labels before you do this.)
11. Choose Copy from the Edit menu.
12. Choose Paste from the Edit menu. This will place a copy of the original triangle in the sketch. Notice that you can resize it independently of the original.
13. Measure the ratio of corresponding sides for the two triangles. You can compute a ratio by selecting a side on one triangle and then Shift-selecting the corresponding side on the other triangle. Then choose Ratio in the Measure menu.

What can you conclude about triangles whose angles are identical?



13. Paste a bunch more copies of the triangle.
14. Play around with some interesting arrangements of these triangles. An example is shown at left.

Day 9: Triangular Possibilities

This day's activity uses some previously prepared sketches to allow students to look at the constraints that exist for angles and lengths of triangles and quadrilaterals. If you situation allows, some students may choose the Sketchpad form of the activity while others work with the off-line form.

Preparation

You will need the following two handouts:

- What's Possible? (Sketchpad version)
- Homework 9: Similar Problems

Load the four sketches on the machines students will use or place them on floppy disks to be used by students.

- Possible Angles
- Possible Lengths
- Quad from Angles
- Quad from Lengths

You will not need any additional materials unless some of your students are going to use the off-line form of the activity.

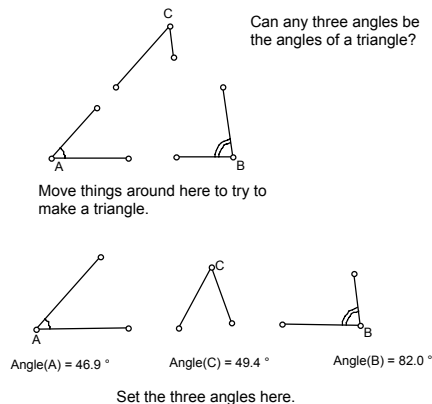
What's Possible?

The Teacher's Notes apply as well as for Sketchpad-based work as for off-line work.

As usual, students are to work in pairs at computers, but for the purposes of reaching consensus on conclusions, they are to work two pairs to a group of four.

Students may need help finding and opening the sketches you have provided for the activity. If you have a demonstration computer, let students know that they will be using it tomorrow to present their findings.

What's Possible?



In this activity you explore what angles and what side lengths are possible for polygons.

For the first two parts, you'll work with a preconstructed sketch. In the third and fourth parts, you'll make your own construction.

Tomorrow, your group will give a presentation on part or all of this activity.

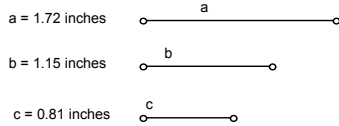
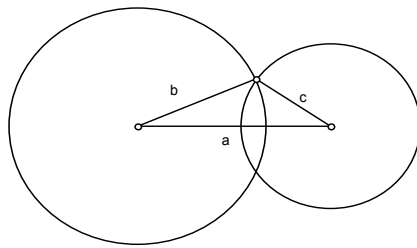
Part I: Angles

Open the sketch "Possible Angles." This sketch lets you set three angles and then try to make a triangle from them.

Can any three angles be the angles of a triangle? Experiment to find the answer to this question. Start with three numbers, say 30° , 50° , and 80° . Can you move the vertices and endpoints of the sides so you get a triangle with sizes?

Try three other values, say 50° , 60° , and 70° . Keep making up numbers and trying them. Keep track of which sets of angles are possible and which are not. What conclusions can you reach about the three angles of a triangle?

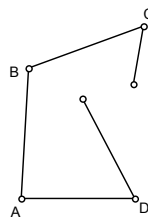
(Note: You may already know the answer to Part I. If so, then state the fact you know.)



Part II: Sides

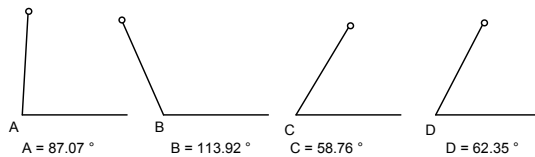
Can any three lengths be the sides of a triangle? Open the sketch "Possible Lengths?" and experiment to find the answer to this question.

As you try different sets of lengths, keep track of which are possible for triangles and which are not. What conclusions can you reach about the three sides of a triangle? (Note: This problem is related to your POW.)



Set the four angles below and then try to drag the points above to make a closed quadrilateral.

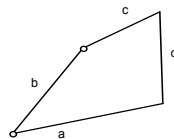
Do any set of four angles work?



Part III: Quadrilaterals

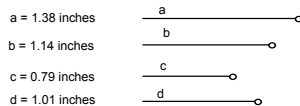
What if we were talking about quadrilaterals (four-sided polygons) instead of triangles? What would be possible for the angles of a quadrilateral?

You may use the sketch "Quad from Angles" to investigate this.



Can any four lengths be used to construct a quadrilateral?

Adjust the lengths below to see find out.



What would be possible for the sides of a quadrilateral? Make similar rules to the ones in parts I and II that apply to quadrilaterals.

You may use the sketch "Quad from Lengths" to investigate this.

Part IV: Other Polygons

What similar rules might apply to all polygons?

Day 13: Outside Angles

In Sketchpad it is incredibly easy to draw polygons, measure exterior angles, and sum them up. The work of the day should go much more easily than based on protractors.

Preparation

You will need the following two handouts:

- Outside Angles
- Homework 13: Shadowy Figures

You will not need any additional materials unless some of your students are going to use the off-line form of the activity.

Outside Angles

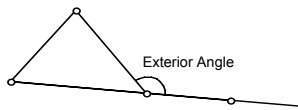
Encourage groups to divide up the problem between the two pairs whenever possible and then pool their findings together. Emphasize that this is often how big jobs are done in the workplace. They should find their own ways to divide up the problem. One common tactic will be for one group to investigate polygons with odd numbers of sides while the other looks at polygons with even numbers of sides.

You may want to demonstrate the construction of an exterior angle before students start. Draw a polygon with the line segment tool. Then extend a side with the ray tool. Point out that you can't measure the exterior angle until you have placed a point on the exterior portion of the ray. Once you have done so, you can select the three points and get the measure of the angle.

If groups finish early, then ask them to expand their investigation to the relationship between interior and exterior angles.

If you have a demonstration computer, let groups know that they can use it tomorrow to present their findings.

Outside Angles



The angle formed by a side of a polygon and the extension of the next consecutive side is called an **exterior angle** of a polygon. The picture to the left shows a triangle and one of its exterior angles.

Investigate exterior angles of polygons. Draw polygons with numbers of sides, use the measuring tools provided by Sketchpad, make tables, and see what you can find. Report your findings on chart paper. Explain why you think your findings are true.

Day 14: Outside Angles Presentations

You may elect to use Sketchpad to help demonstrate what is going on in the discussion of Homework 13, Shadowy Figures.

Many groups will elect to present their findings on Outside Angles using Sketchpad, and you can demonstrate that the sum of the exterior angles of any polygon is 360° using a powerful visual demonstration.

Discussion of Homework 13: Shadowy Figures

On the disk that accompanies this supplement, you'll find sketches that demonstrate each part of this homework. Use these at your discretion, after student volunteers share their work to help students understand what is going on. By dragging the rope for problem 2, students can see that the shadows of the knotted segments must be unequal except in the case where the rope is parallel to the ground. This may help prepare students for the discussion based on the diagram shown in the teacher notes for the unit.

Outside Angles Presentations

If you have access to a classroom demonstration computer, groups can present their findings using Sketchpad.

The following visual demonstration that the sum of the exterior angles of a polygon equals 360° is very worthwhile. You can either do it by following the instructions below, or by opening the sketch, "Exterior Angles Demo" included with the supplement.



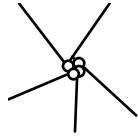
Using the ray tool, draw a pentagon as shown at left.

Select one of the points and choose the Mark Center command from the Transform menu.



Use the dilation tool.

Select everything in the sketch.



Drag any point except the point you marked as a center. Draw attention to the exterior angles as you drag toward the marked center. Students should be able to explain why their sum must be 360° .

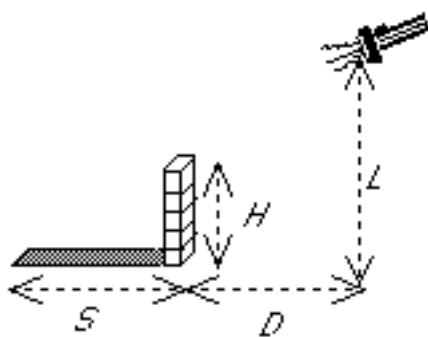
Day 17: A Shadow of a Doubt

Going back to the Sketchpad model they constructed on day 4c, students relate the geometric model to an algebraic model using similar triangles and algebra. They can express the algebraic relation directly in Sketchpad and test it against the geometric model's predictions.

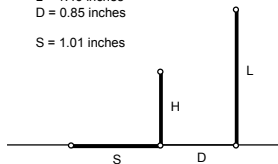
A Shadow of a Doubt

Make sure students can open up the shadow model sketch they last used on day 4c. Explain that today they will use this sketch to get a formula for the length of the shadow when they know the height of the object, the distance of the light from the object, and they distance of the light above the ground.

A Shadow of a Doubt



H = 0.79 inches
L = 1.46 inches
D = 0.85 inches
S = 1.01 inches



Can your group predict the length of a shadow without a shadow of a doubt? Recall the following variables, as shown in the diagrams:

- **L** = the distance from the Light source to the ground
- **D** = the Distance *along the ground* from the light source to the object casting the shadow
- **H** = the Height of the object casting the shadow
- **S** = the length of the Shadow

Work with the shadows model you constructed in the Modeling Shadows investigation near the beginning of this unit.

1. Find a pair of similar triangles. Explain why these triangles must be similar.
2. Find *another* pair of similar triangles. (Hint: you may have to construct an additional line segment.)

3. Use your knowledge of similar triangles to write two different equations that express a relationship among these four variables. (One equation will come from each pair of similar triangles.)
4. Verify that your equations hold by using Sketchpad to compute each side of each equation. For example, if one side of an equation is H/L , select the measurement for H and the measurement for L and use the Calculate command to compute H/L . Then do the same for the other side of the equation. Drag things around in your sketch to see if the equality holds.
5. Students at Mystery High did some experiments set up like the one above. The problem is that, in each case, they did not write down the length of the shadow. Find the length of the shadow in each situation below.
 - a. $L = 11$, $H = 5$, and $D = 12$
 - b. $L = 15$, $H = 5$, and $D = 12$
 - c. $L = 15$, $H = 5$, and $D = 60$
4. Write an equation that shows how to find the length of a shadow when L , H , and D are given.

*Day 18: POW Presentations and Similar Areas*Finishing A Shadow of A Doubt

As described in the unit notes, there may not have been enough time to finish this investigation and any remaining time should be used to complete it and discuss it. A classroom demonstration computer will facilitate the discussion.

POW 18: Billiard Bankshots

If students have access to Sketchpad outside of class, you can suggest that some of them undertake to do this POW with Sketchpad.

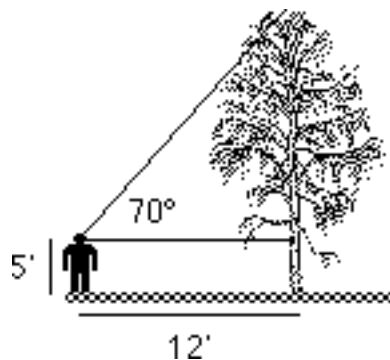
Day 19: Finding a Function

If all has gone well, you will have already accomplished the goal of this day on day 17 or 18! This should be a day that you gain back from the three that were added near the beginning of the unit.

Day 20: The Return of the Tree and Introducing Trigonometry

Two days' work in the regular unit are here combined into one as we hope to capitalize on some of the students' investment in learning how to investigate problems with Sketchpad. As in the regular unit, we present a new way of measuring the height of a tree that leads to trigonometric functions.

Probably the most common method of using Sketchpad to solve the tree problem will be to construct a scale model. Students will likely find that a scale model in which 1 foot = 1 inch, or even 1 foot = 1 centimeter, will not fit on the computer screen. As a result, they'll have to come up with some other scale factor such as 4 feet = 1 inch. Notice how this scaling factor is just the ratio of sides of the real life problem to the model!

The Return of the Tree

Woody has found a new way to measure a tree. (I'll bet you can't wait to hear!)

First he measures the height from the ground to his eye. That's 5 feet.

Then he uses a protractor to measure the angle between the horizontal and his line of sight up to the top of the tree. That's 70°.

Finally, he measures his distance from the tree. That's 12 feet.

That's all the information that Woody needed. Using these measurements, he figured out how tall the tree was. So you, with the help of Sketchpad and your understanding of similarity, should be able to figure it out also.

It shouldn't take long to model the problem and find an answer. Discuss methods students used. In the discussion ask them to find out an angle for which the height of the tree equals the distance from

Woody to the tree. And, what is the angle when the tree is just as tall as Woody?

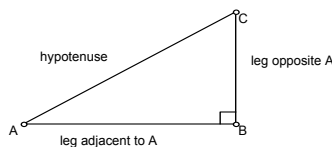
Introducing Trigonometry

The sides of right triangles have special names.

The longest side is called the **hypotenuse**; it is always opposite (furthest from) the right angle.

The other two sides are called the **legs**. Each has a specific name, but the name depends on which non-right angle one looks at. The leg closest to the chosen angle is called the **adjacent leg**. The side furthest from the chosen angle is the **opposite leg**.

For example, $\triangle ABC$ below is a right triangle with a right angle at vertex B . If we focus on the non-right angle A , then segment \overline{BC} becomes the *opposite leg* and segment \overline{AB} becomes the *adjacent leg*.



Use Sketchpad to construct a right triangle such as the one above. (Make sure that the right angle is *constructed* using the Perpendicular Line command in the Construct menu.)

Based on this terminology, the trigonometric ratios are defined as follows:

The **sine** of angle A is the ratio of the length of the side *opposite* angle A divided by the length of the **hypotenuse**. The sine of angle A is abbreviated by "sin A ."

$$\sin A = \frac{\text{leg opposite } A}{\text{hypotenuse}}$$

(or just: $\frac{\text{opposite}}{\text{hypotenuse}}$)

The **cosine** of angle A is the ratio of the length of the side *adjacent* to angle A divided by the length of the **hypotenuse**. The cosine of angle A is abbreviated by "cos A ."

$$\cos A = \frac{\text{leg adjacent to } A}{\text{hypotenuse}}$$

(or just: $\frac{\text{adjacent}}{\text{hypotenuse}}$)

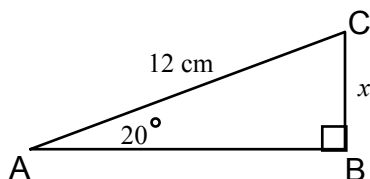
The **tangent** of angle A is the ratio of the length of the side *opposite* angle A divided by the length of the leg *adjacent* to angle A. The tangent of angle A is abbreviated by "tan A."

$$\tan A = \frac{\text{leg opposite A}}{\text{leg adjacent to A}}$$

(or just: $\frac{\text{opposite}}{\text{adjacent}}$)

Angle(CAB) = 34.30 °
Sin[Angle(CAB)] = 0.56

1. In your sketch, measure angle A. Select the measurement, and use the Calculate command from the Measure menu to compute the sine of that angle.
2. Are you wondering how all this will improve the quality of your life? Well, these ratios make finding lengths of sides of right triangles much easier. For example, in the triangle below, the side opposite the 20° angle has length x , and the hypotenuse has length 12 cm. You can use your calculator to find $\sin 20^\circ$. This number will be the ratio of x to 12. You can then find x as shown:



$$\sin 20^\circ = \frac{x}{12} .$$

On the calculator, we find that $\sin 20^\circ$ is approximately .34202, so:

$$\frac{x}{12} \approx .34202.$$

("♠" means "is approximately equal to.")

$$x \approx 12(.34202)$$

$$x \approx 4.1042$$

So the side labeled x must be about 4 cm long.

Length(Segment hypotenuse) = 2.48 inches
Length(Segment opposite) = 1.21 inches
Length(Segment hypotenuse)*Sin[Angle(BAC)] = 1.21 inches

3. In your sketch, measure the length of the hypotenuse and the length of the side opposite angle A .

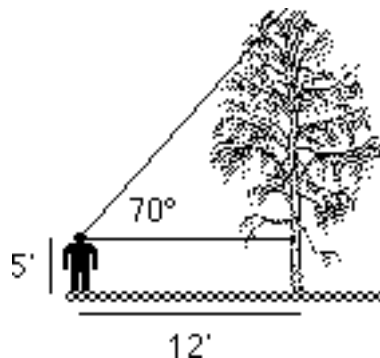
Using the arrow tool, select the calculation of the sine and the measurement of the hypotenuse.

Use the Calculate command to calculate the sine of the angle times the length of the hypotenuse.

Compare the result with the measured length of the opposite side, especially as you change angle A by dragging the vertices of the triangle.

4. Find the height of Woody's tree using trigonometry.

(No, the word "trigonometry" has nothing to do with trees.)



Day 21: Sin, Cos, and Tan Revealed

This day's activity was combined with the previous day's on the theory that Sketchpad makes it very easy to see what is going on with trigonometric functions.

Sketches and Scripts

Homework 4c

Copy Angles Script

Possible Angles

Possible Lengths

Quad from Angles

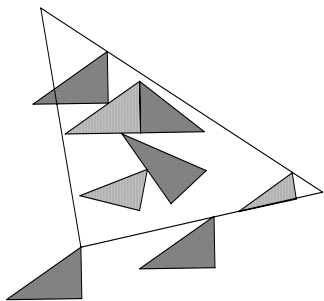
Quad from Lengths

Homework 13, Shadowy Figures

Exterior Angles Demo

Worksheets and Homeworks

Exploring Right Triangles



Work in pairs on this tutorial to learn how to use The Geometer's Sketchpad as a tool that will help you investigate the shadows problem. One person can read the tutorial while the other carries out the instructions on the computer. Trade roles several times so both of you get a chance to try using the computer.

<Say something here about what they should prepare for discussion with the rest of the class.>

Any Old Triangle



Click once on the segment tool. When you move the mouse pointer into the drawing area of the window, notice that it becomes a crosshair. You draw a segment by holding down the mouse button in one spot and dragging to another spot before releasing.



Use the segment tool to draw a triangle.



Click once on the arrow tool. Move the mouse pointer over one of the vertices, hold the button down, and drag the vertex. Try dragging the other two vertices. Finally, try dragging the sides of the triangle.

Label each of the statements below true or false.

- ___ 1. When you drag a vertex of the triangle, the side opposite that vertex does not change length.
- ___ 2. When you drag a side of the triangle, all three sides change length.
- ___ 3. You can drag the triangle so that it is a right triangle.

How to Label Points and Segments



Click once on the label tool. Notice how the mouse pointer becomes a hand when you move it into the drawing area. When you move the finger on top of something you can label—a point, line, or circle—it highlights. When the hand is highlighted, click the mouse button. The object you clicked on will get a label.



To make the label go away, click on the object with the label tool again. One click to show the label, another click to hide it.



Point the finger at a label. Notice that the hand gets a little letter “A” in it. Hold down the mouse button and you can move the label around. Notice, though, that you can’t move the label very far from its object.



Double-click on a label with the hand’s finger. You will get a dialog box with which you can change the label to whatever you want. Click the OK button when you have what you want.

Experiment with labels in order to decide which of the following statements are true and which are false:

- ___ 4. Line segments can have labels.
- ___ 5. When a point moves, its label moves with it.
- ___ 6. Once you have changed a label, you can’t hide that label.

How to Undo What You Did

Edit	Display	Construct	Tr
Undo	Construct	Point	⌘Z
Redo			⌘R
Cut			⌘H
Copy			⌘C
Paste			⌘V

The Edit menu contains the Undo command. Each time you select it, the last thing you did gets undone. You can keep “undoing” all the way back to the beginning of your work. You can also use the Redo command to redo something you just undid. Got it?

The absolute best way to fix most mistakes in Sketchpad is to keep undoing until the mistake is gone.

True or false?

- ___ 7. When you drag a point, the Undo command does *not* put the point back where it was.

How to Construct a Right Triangle

The triangle you made could be *any* triangle because you could drag the vertices anywhere you wanted. Now you'll learn how to construct a triangle that will always stay a *right* triangle, a triangle with a 90° angle in it.

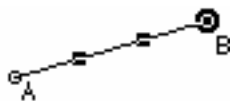


Choose New Sketch from the File menu. This will give you a blank sketch in which you can begin a new construction.

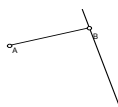


Using the segment tool, draw a segment. Label its endpoints *A* and *B*.

Next you'll construct a line through point *B* perpendicular to the segment.



Using the arrow tool, select point *B*. Now hold down the Shift key and click on the segment. You should see that both the point and the segment are selected, as shown at left.

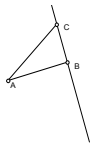


Choose the Perpendicular Line command from the Construct menu. You should get a line through point *B* perpendicular to segment *AB*.

Try dragging various points around and notice how the line through point *B* always remains perpendicular to segment *AB*.

True or false?

- ___ 8. Dragging point *A* does not change the orientation (slope) of the perpendicular line through *B*.
- ___ 9. Dragging the perpendicular line drags point *A*.



Using the segment tool, draw a segment from somewhere on the perpendicular line to point A.

Using the label tool, label the new point C.

Using the arrow tool, select the perpendicular line.

Issue the Hide Line command from the Display menu. This should make the perpendicular line disappear.

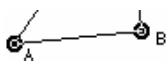
Using the segment tool, connect points B and C to complete your right triangle.

True or false?

___ 10. Point C can only be dragged along the hidden perpendicular line.

___ 11. Dragging point A changes the length of segment BC.

How to Measure Distances and Lengths



Using the arrow tool select point A (without holding down the Shift key). Then hold down the Shift key and select point B. Both points should highlight as shown at left.



If the two points are properly selected, you should be able to issue the Distance command from the Measure menu.

Distance(A to B) = 1.00 inches

The command's result is to place the desired measurement in the sketch as shown at left.

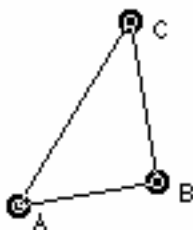
Length(Segment m) = 1.00 inches

You can also measure the length of a segment. To do so, select a segment with the arrow tool. Then choose the Length command from the Measure menu.

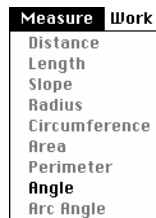
Decide about the truth or falsity of each of the following statements by taking some measurements of parts of your right triangle.

- ___ 12. No matter how you drag the points, the distance between points A and C stays greater than the distance between A and B .
- ___ 13. The length of segment BC can never be made zero.

How to Measure Angles



An angle measurement requires that you select three points, the middle of which is the vertex of the angle whose measure you want. To measure the angle at vertex B , select point A with the arrow tool, then hold down the Shift key and select points B and C in that order.



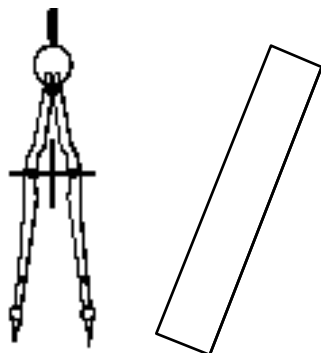
With the three points properly selected, you should be able to issue the Angle command from the Measure menu, resulting in the measurement appearing in the sketch window.

Measure the angles at the other two vertices of your right triangle.

By dragging parts of your triangle around and observing your measurements, answer each of the following questions.

14. What is the sum of the measure of angle A and the measure of angle B ? Is this sum always the same?
15. What is the measure of angle A when the length of AB equals the length of BC ?
16. When the length of AC is 2 inches and the length of BC is 1 inch, what is the measure of angle A ?

If you finish early, come up with some statements that seem to be true about your right triangle. Be prepared to share these with the rest of the class.

Homework 4a: Euclid's Tools

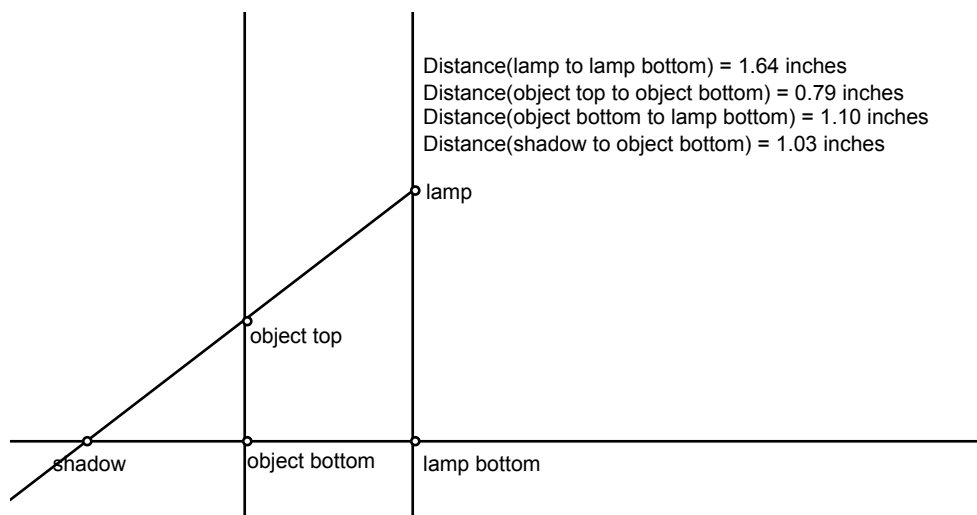
You need to have a sharp pencil, a compass and a straightedge to do this homework.

For each item below decide whether you can use your three tools to accomplish the construction. (Note that you are not allowed to use the straightedge as a ruler!) If you can do it, show the result and explain in words how you did it. If you cannot do it, explain why the methods you tried did not work.

1. Given two points, construct a line passing through both of them.
2. Given two points, construct a circle whose center is at one point and which passes through the second point.
3. Given two points, construct a point exactly halfway between them.
4. Construct a right angle.
5. Construct two parallel lines. (Using the opposite parallel edges of the straightedge is *not* a valid construction!)

Modeling Shadows

Your goal in this activity is to construct a model of the shadows experiment using The Geometer's Sketchpad. You will use the model to gather data that will supplement the data you gather from the experiment.



The figure above shows what you want. But beware, good looks are not sufficient. The parts of your model must *behave* well, too.



Hint: You will almost certainly need to use the ray tool.

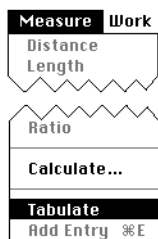
Another hint: If your model doesn't behave the way you want it to, don't be afraid to start over again with a new sketch.

Here is checklist of questions to use to verify that your model behaves well. *Write down an answer to each question.*

1. What distances change when you drag the lamp?
2. What distances change when you drag the lamp bottom?
3. What distances change when you drag the object top?
4. What distances change when you drag the object bottom?

How to Make a Table of Measurements

Collecting measurements in a table will save you from having to copy numbers onto paper.



Using the arrow tool, select one of the measurements. Hold down the shift key and select the other measurements.

Choose Tabulate from the Measure menu. You should get a table similar to the one below.

Distance(lamp to lamp bottom)	1.64
Distance(object top to object...	0.79
Distance(object bottom to lam...	1.10
Distance(shadow to object bot...	1.03



Choose the label tool. With the tip of the tool's finger, double-click on a row label. Type in a new row label that will be easier to understand.



When you are finished changing the row labels, your table should look similar to the one at left.

Using the arrow tool, select the main part of the table, the part where the numbers are.



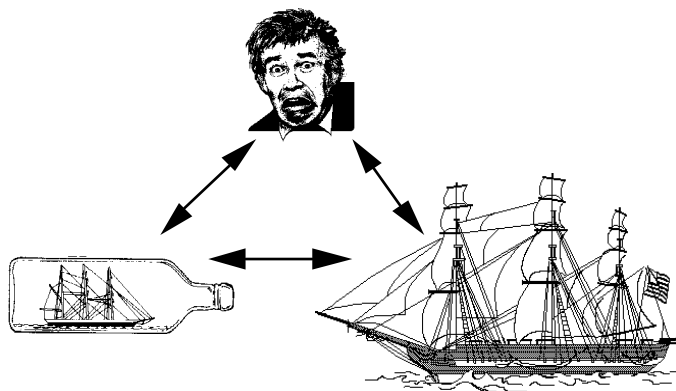
Choose the Flip Direction command from the Measure menu. This should orient your table with the labels at the top of columns as shown at left.

Change something, such as the height of the object, in your sketch. Now double-click on the main part of the table. Notice that a new row of entries gets added to your table.

If You Have Time

Experiment with your model of the shadows experiment. Write down some observations about how the model works.

Important! Be sure to save your sketch on disk. You will need to use it on the next day to gather data.

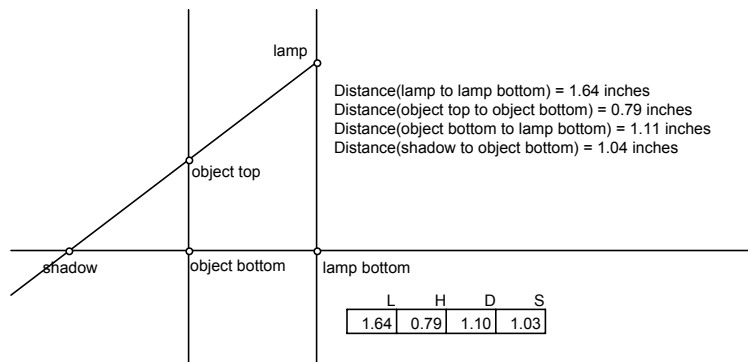
Homework 4b: Modeling Madness

Each thing listed below can be considered a model of something. For each, answer the following questions:

- a. What is it a model of?
 - b. Under what circumstances might you prefer using the model to using the real thing? And why?
 - c. List two questions you would be able to answer using the model.
1. A road map of the United States.
 2. A mockup of an airplane wing placed in a wind tunnel.
 3. An architect's plans for a skyscraper.
 4. A flight simulator.
 5. A tailor's pattern for a jacket.
 6. A rug diagram for the game of Fig.

Shadows on the Screen

Open the sketch you saved at the end of the last class period. Presumably it looks similar to the one shown below.



Your job here is to perform an “experiment” using your Sketchpad model of the shadows situation. You will gather and analyze data for the length of the shadow as one of the other three variables changes.

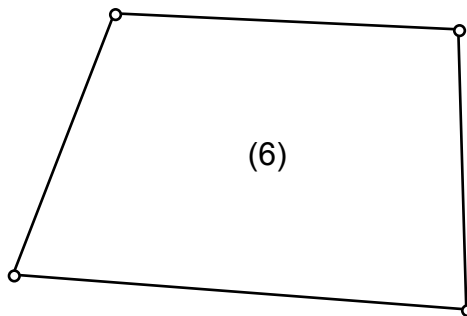
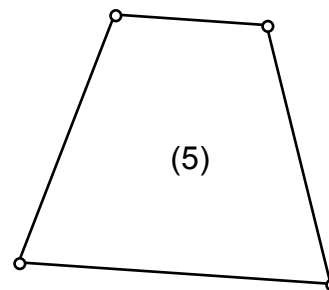
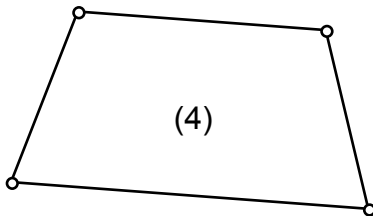
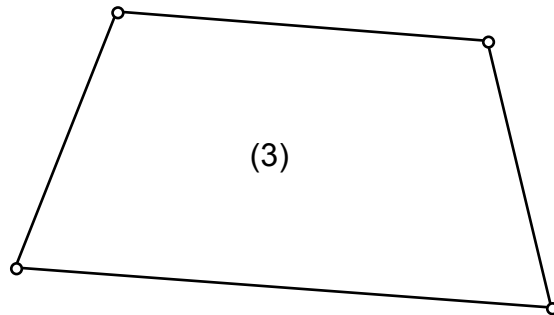
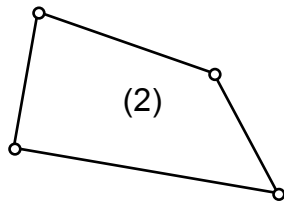
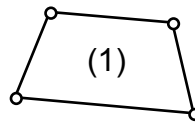
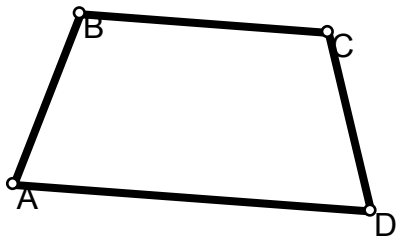
- Choose one of the variables, D , H , or L , to change while holding the other two constant. Take at least ten measurements. (To add an entry to a table, double-click the numbers in the table.)
- On graph paper make a plot of your data. If, for example, you chose to vary D , then put D on the horizontal axis and S on the vertical axis.
 - In what ways does it look *similar* to the graphs produced from the physical data?
 - In what ways does your graph look *different* from the graphs produced from the physical data?
- Attempt to fit an equation through your data points. You can use any of the three methods discussed on the *Looking for Equations* worksheet.
- Repeat steps 1–3 with a different variable.

Homework 4c: Getting One from Another

Each of the seven numbered quadrilaterals below was produced by doing something to quadrilateral $ABCD$ in the upper left corner. For each,

- a) describe how it was produced from $ABCD$, and
- b) describe how you decided this.

(You'll need a ruler and protractor.)



How to Copy Lengths and Angles – a Tutorial



It often happens as you construct something with Sketchpad that you would like one length to stay equal to another or one angle to stay the same as another. This tutorial will show you how to do that.

How to Copy a Length



Using the segment tool, draw a segment.



Label the end points *A* and *B*.

Using the point tool, place a point in your sketch.

Label this point *C*.



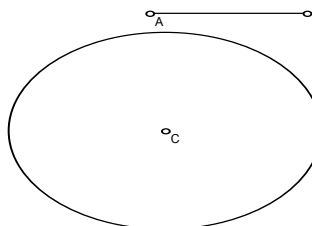
Using the arrow tool, select point *C*.

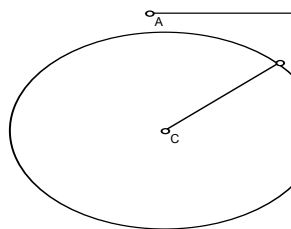
Holding down the Shift key, select the segment between points *A* and *B*.



Choose Circle by Center + Radius from the Construct menu. The result should be a circle centered at *C* with radius equal to the length of *AB*.

Construct	Transform	Mea
Point On Object		
Point At Intersection	⌘I	
Point At Midpoint	⌘M	
Segment		⌘L
Perpendicular Line		
Parallel Line		
Angle Bisector		
Circle By Center+Point		
Circle By Center+Radius		
Polygon Interior	⌘P	
Circle Interior		
Construction Help...		





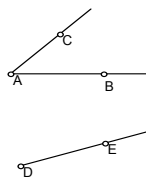
Using the segment tool, draw a segment from point C to any place on the circumference of the circle.

Using the arrow tool, select the circle.

Choose Hide Circle from the Display menu.

Drag points A and B . Notice that the length of the new segment remains equal to the length of AB , but its direction is independent.

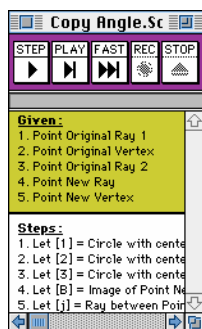
How to Copy an Angle



Use the Ray tool to draw a angle BAC as shown at left.

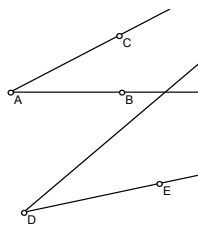
Draw a ray DE as shown below and to the left. The task is to duplicate angle BAC at vertex D .

Choose the Open command from the file menu and open the script entitled CopyAnglesScript.



The script you have opened will automatically copy an angle. Notice that the top part of the script window contains a list of five "givens." These are the three points that define the original angle plus the two points that define one side of the new angle.

Using the arrow tool, select point B . Holding down the Shift key, click on points A , C , D , and E in that order.



Press the Fast button in the top bar of the script window.

A new ray should appear from point D .

Drag point C . Notice how the angle at D remains equal to angle BAC .

Is There a Counterexample?



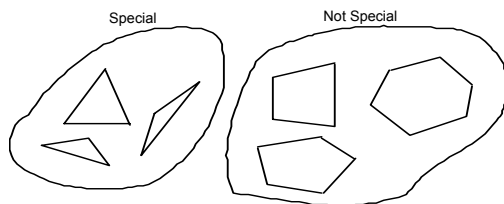
Use Sketchpad to attempt to construct counterexamples to each of the statements below. If you are able to find one counterexample, go on and find a second.

Explain why your examples disprove the given statement.

If you do not think a counterexample exists, then explain why you think so.

1. If two polygons have their corresponding angles equal, then the polygons are similar.
2. If two polygons have their corresponding sides proportional, then the polygons are similar.
3. If a triangle has two equal sides, then it will also have two equal angles.

Why are Triangles Special?

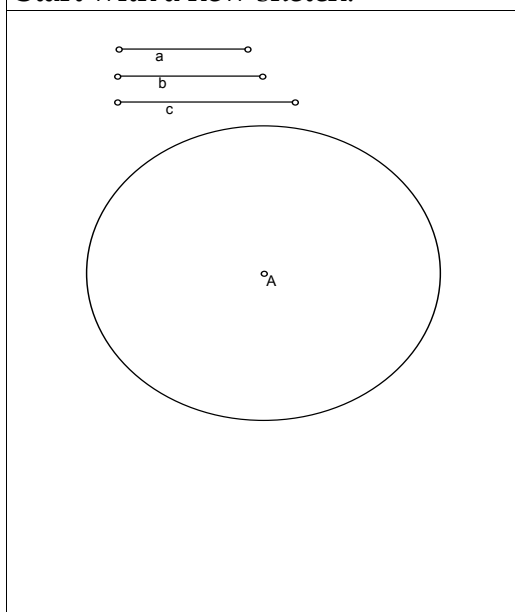


Triangles have some properties that separate them from your average polygon. In this investigation you will find out about these properties and relate them to similarity.

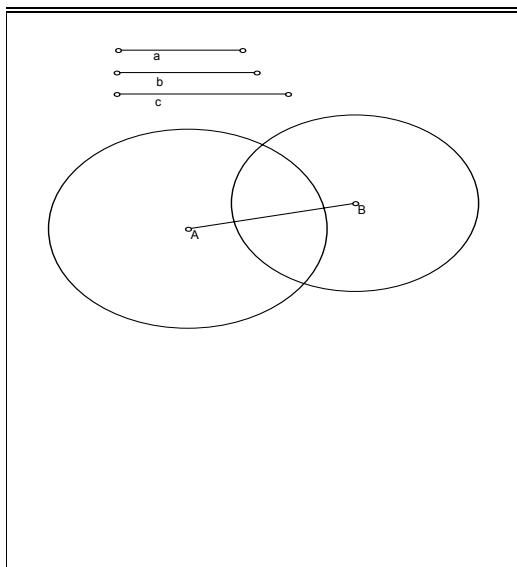
I. Constructing a Triangle with Fixed Length Sides

Normally, a triangle you draw with Sketchpad has sides like rubber bands. Here you learn how to draw one with fixed length sides.

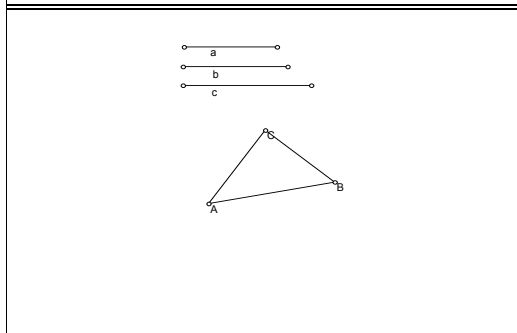
Start with a new sketch.



1. Place three line segments in your sketch. These will be the lengths of the sides of your triangle. Label them a , b , and c .
2. Place a point in your sketch to serve as the first vertex of the triangle. Label it A .
3. Select point A and segment c (using the Shift key).
4. From the Construct menu choose Circle by Center+ Radius.



5. Draw a line from A to some point on the circumference of the circle.
6. Hide the circle.
7. Label the point on the circumference B .
8. Construct a circle with center A and radius b .
9. Construct a circle with center B and radius a .

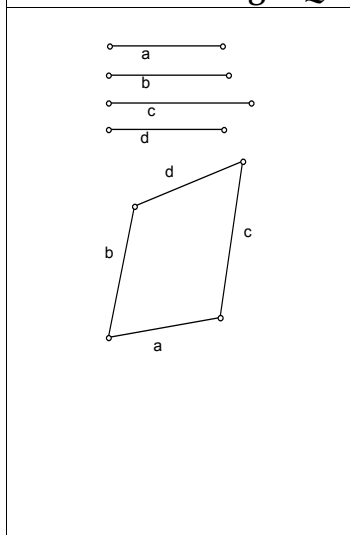


10. Construct a point at the intersection of the two circles.
11. Label the point C .
12. Hide the two circles.
13. Draw segments AC and BC .

Drag the points in your sketch so that you can get a sense of what triangles are possible and what ones are not.

Write five observations based on your sketch. Be prepared to share these with the rest of the class.

II. Constructing a Quadrilateral with Fixed Length Sides



Using the same techniques that you used to construct a triangle with fixed length sides, construct a quadrilateral with fixed length sides. Your sketch will look similar to the one shown at left.

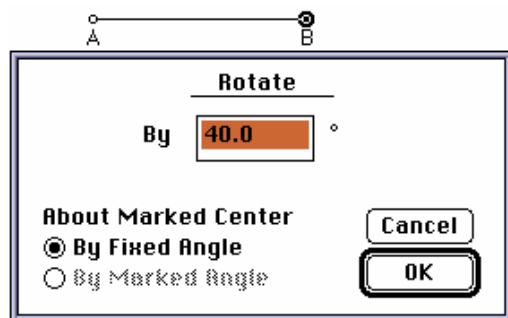
What differences do you observe when you compare your triangle sketch with your quadrilateral sketch?

Suppose two triangles have their corresponding sides proportional. What does that imply about their angles?

Must two triangles with corresponding sides proportional be similar or could they have different angles? How about for quadrilaterals? What do you think would be true for polygons with more sides?

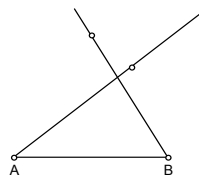
III. Constructing a Triangle with Fixed Angles

In the previous activity you constructed polygons whose side lengths were rigid. Now you'll construct a triangle by fixing the angles and seeing what is left free to change.



1. Draw line segment AB .
2. Select point A and choose the Mark Center "A" command from the Transform menu.
3. Select point B and choose the Rotate command from the Transform menu.

4. In the dialog box that comes up, specify 40 degrees as the rotation angle and press OK.



5. Use the ray tool connect point A to the rotated point B .
6. Repeat steps 2-5 using point B as the center of a *negative* 60 degree rotation of point A . Your sketch should look similar to the one at left.

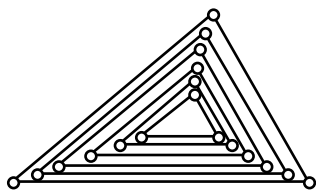
7. Place a point at the intersection of the two rays. Label it C .
8. Hide the two rotated points and the two rays.
9. Draw segments AC and BC .

Experiment with your triangle. What things can you do to it? What things can you *not* do to it?

10. Using the arrow tool select the entire triangle, ABC . (It will be helpful if you hide the point labels before you do this.)
11. Choose Copy from the Edit menu.

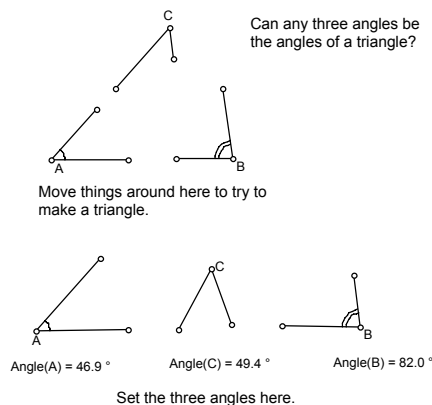
12. Choose Paste from the Edit menu. This will place a copy of the original triangle in the sketch. Notice that you can resize it independently of the original.
13. Measure the ratio of corresponding sides for the two triangles. You can compute a ratio by selecting a side on one triangle and then Shift-selecting the corresponding side on the other triangle. Then choose Ratio in the Measure menu.

What can you conclude about triangles whose angles are identical?



13. Paste a bunch more copies of the triangle.
14. Play around with some interesting arrangements of these triangles. An example is shown at left.

What's Possible?



In this activity you explore what angles and what side lengths are possible for polygons.

For the first two parts, you'll work with a preconstructed sketch. In the third and fourth parts, you'll make your own construction.

Tomorrow, your group will give a presentation on part or all of this activity.

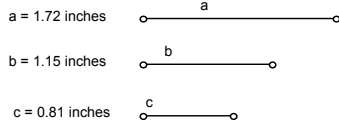
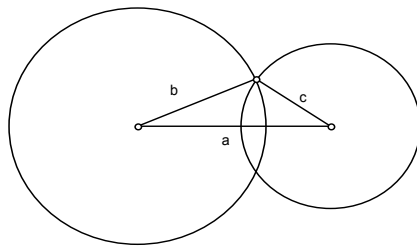
Part I: Angles

Open the sketch "Possible Angles." This sketch lets you set three angles and then try to make a triangle from them.

Can any three angles be the angles of a triangle? Experiment to find the answer to this question. Start with three numbers, say 30° , 50° , and 80° . Can you move the vertices and endpoints of the sides so you get a triangle with sizes?

Try three other values, say 50° , 60° , and 70° . Keep making up numbers and trying them. Keep track of which sets of angles are possible and which are not. What conclusions can you reach about the three angles of a triangle?

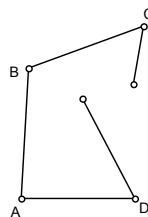
(Note: You may already know the answer to Part I. If so, then state the fact you know.)



Part II: Sides

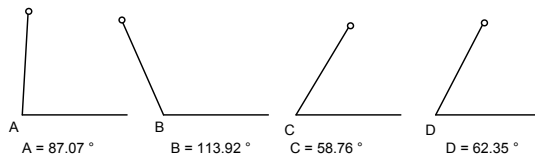
Can any three lengths be the sides of a triangle? Open the sketch "Possible Lengths?" and experiment to find the answer to this question.

As you try different sets of lengths, keep track of which are possible for triangles and which are not. What conclusions can you reach about the three sides of a triangle? (Note: This problem is related to your POW.)



Set the four angles below and then try to drag the points above to make a closed quadrilateral.

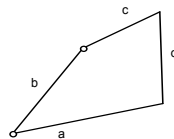
Do any set of four angles work?



Part III: Quadrilaterals

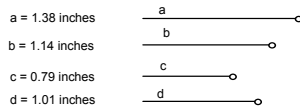
What if we were talking about quadrilaterals (four-sided polygons) instead of triangles? What would be possible for the angles of a quadrilateral?

You may use the sketch "Quad from Angles" to investigate this.



Can any four lengths be used to construct a quadrilateral?

Adjust the lengths below to see find out.



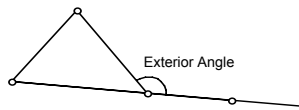
What would be possible for the sides of a quadrilateral? Make similar rules to the ones in parts I and II that apply to quadrilaterals.

You may use the sketch "Quad from Lengths" to investigate this.

Part IV: Other Polygons

What similar rules might apply to all polygons?

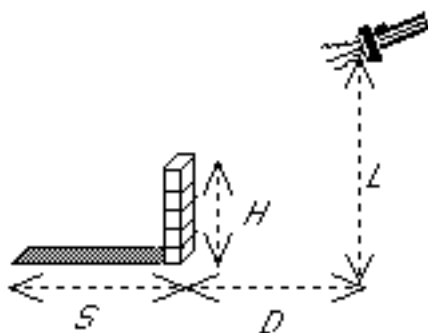
Outside Angles



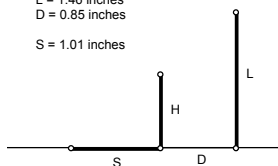
The angle formed by a side of a polygon and the extension of the next consecutive side is called an **exterior angle** of a polygon. The picture to the left shows a triangle and one of its exterior angles.

Investigate exterior angles of polygons. Draw polygons with numbers of sides, use the measuring tools provided by Sketchpad, make tables, and see what you can find. Report your findings on chart paper. Explain why you think your findings are true.

A Shadow of a Doubt



H = 0.79 inches
 L = 1.46 inches
 D = 0.85 inches
 S = 1.01 inches



Can your group predict the length of a shadow without a shadow of a doubt? Recall the following variables, as shown in the diagrams:

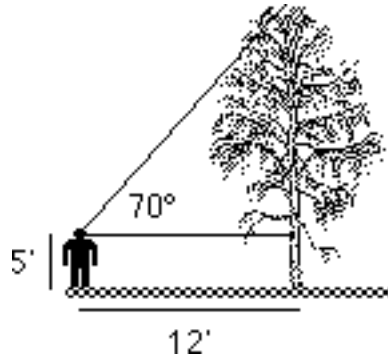
- **L** = the distance from the **L**ight source to the ground
- **D** = the **D**istance *along the ground* from the light source to the object casting the shadow
- **H** = the **H**eight of the object casting the shadow
- **S** = the length of the **S**hadow

Work with the shadows model you constructed in the Modeling Shadows investigation near the beginning of this unit.

1. Find a pair of similar triangles. Explain why these triangles must be similar.
2. Find *another* pair of similar triangles. (Hint: you may have to construct an additional line segment.)
3. Use your knowledge of similar triangles to write two different equations that express a relationship among these four variables. (One equation will come from each pair of similar triangles.)
4. Verify that your equations hold by using Sketchpad to compute each side of each equation. For example, if one side of an equation is H/L , select the measurement for H and the measurement for L and use the Calculate command to compute H/L . Then do the same for the other side of the equation. Drag things around in your sketch to see if the equality holds.

5. Students at Mystery High did some experiments set up like the one above. The problem is that, in each case, they did not write down the length of the shadow. Find the length of the shadow in each situation below.
 - a. $L = 11$, $H = 5$, and $D = 12$
 - b. $L = 15$, $H = 5$, and $D = 12$
 - c. $L = 15$, $H = 5$, and $D = 60$
4. Write an equation that shows how to find the length of a shadow when L , H , and D are given.

The Return of the Tree



Woody has found a new way to measure a tree. (I'll bet you can't wait to hear!)

First he measures the height from the ground to his eye. That's 5 feet.

Then he uses a protractor to measure the angle between the horizontal and his line of sight up to the top of the tree. That's 70°.

Finally, he measures his distance from the tree. That's 12 feet.

That's all the information that Woody needed. Using these measurements, he figured out how tall the tree was. So you, with the help of Sketchpad and your understanding of similarity, should be able to figure it out also.

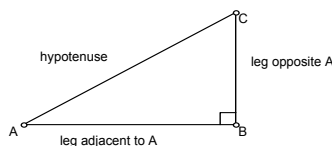
Introducing Trigonometry

The sides of right triangles have special names.

The longest side is called the **hypotenuse**; it is always opposite (furthest from) the right angle.

The other two sides are called the **legs**. Each has a specific name, but the name depends on which non-right angle one looks at. The leg closest to the chosen angle is called the **adjacent leg**. The side furthest from the chosen angle is the **opposite leg**.

For example, $\triangle ABC$ below is a right triangle with a right angle at vertex B. If we focus on the non-right angle A, then segment \overline{BC} becomes the *opposite leg* and segment \overline{AB} becomes the *adjacent leg*.



Use Sketchpad to construct a right triangle such as the one above. (Make sure that the right angle is *constructed* using the Perpendicular Line command in the Construct menu.)

Based on this terminology, the trigonometric ratios are defined as follows:

The **sine** of angle A is the ratio of the length of the side *opposite* angle A divided by the length of the **hypotenuse**. The sine of angle A is abbreviated by "sin A."

$$\sin A = \frac{\text{leg opposite A}}{\text{hypotenuse}}$$

(or just: $\frac{\text{opposite}}{\text{hypotenuse}}$)

The **cosine** of angle A is the ratio of the length of the side *adjacent* to angle A divided by the length of the **hypotenuse**. The cosine of angle A is abbreviated by "cos A."

$$\cos A = \frac{\text{leg adjacent to A}}{\text{hypotenuse}}$$

(or just: $\frac{\text{adjacent}}{\text{hypotenuse}}$)

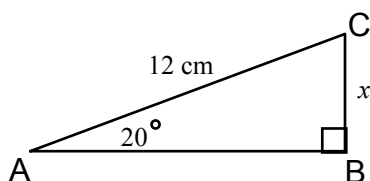
The **tangent** of angle A is the ratio of the length of the side *opposite* angle A divided by the length of the leg *adjacent* to angle A. The tangent of angle A is abbreviated by "tan A."

$$\tan A = \frac{\text{leg opposite A}}{\text{leg adjacent to A}}$$

(or just: $\frac{\text{opposite}}{\text{adjacent}}$)

Angle(CAB) = 34.30 °
Sin[Angle(CAB)] = 0.56

1. In your sketch, measure angle A. Select the measurement, and use the Calculate command from the Measure menu to compute the sine of that angle.
2. Are you wondering how all this will improve the quality of your life? Well, these ratios make finding lengths of sides of right triangles much easier. For example, in the triangle below, the side opposite the 20° angle has length x , and the hypotenuse has length 12 cm. You can use your calculator to find $\sin 20^\circ$. This number will be the ratio of x to 12. You can then find x as shown:



$$\sin 20^\circ = \frac{x}{12} .$$

On the calculator, we find that $\sin 20^\circ$ is approximately .34202, so:

$$\frac{x}{12} \approx .34202.$$

("♠" means "is approximately equal to.")

$$x \approx 12(.34202)$$

$$x \approx 4.1042$$

So the side labeled x must be about 4 cm long.

Length(Segment hypotenuse) = 2.48 inches
Length(Segment opposite) = 1.21 inches
Length(Segment hypotenuse)*Sin(Angle(BAC)) = 1.21 inches

3. In your sketch, measure the length of the hypotenuse and the length of the side opposite angle A .

Using the arrow tool, select the calculation of the sine and the measurement of the hypotenuse.

Use the Calculate command to calculate the sine of the angle times the length of the hypotenuse.

Compare the result with the measured length of the opposite side, especially as you change angle A by dragging the vertices of the triangle.

4. Find the height of Woody's tree using trigonometry.

(No, the word "trigonometry" has nothing to do with trees.)

