

My school, Ridley Middle School, in Delaware County, Pennsylvania, adopted the Connected Mathematics Project for the 2000-2001 school year in grade six. Grades seven and eight are to follow next year. As a mathematics educator, I was extremely excited when I found out the district was to adopt this program as I feel it embodies the National Council of Teachers of Mathematics' Curriculum and Evaluation Standards for School Mathematics. In this paper, I will show how the Connected Mathematics Project is directly aligned with the communication standard for the middle grades. Obviously, there are many modes of communication in education. Connected Math explores these various modes and gives students many ways to demonstrate mathematical thinking. I will also focus on how well Connected Math focuses on writing in the math classroom. Over the past decade and a half, writing in the mathematics classroom has been a very popular math education topic in published journals and books and at various workshops and conferences.

The first major point highlighted by the "Mathematics as Communication" standard is that "the study of mathematics should include opportunities to communicate so that students can model situations using oral, written, concrete, pictorial, graphical and algebraic methods." Connected Math has students work in groups or individually to model many different situations for the main purpose of discovering mathematical concepts or important applications of mathematical concepts. The program uses all of the described modeling methods but in each instance, students are asked to describe or explain their results. Connected Math's books are broken into "Investigations" instead of chapters and the subdivision of the investigations are called "Problems". The "Problems" consist of the modeling situations described above. During each problem, students must use writing to make cohesive statements about what they have just learned. This is an integral part of each investigation, as the teacher no longer stands in front of

the class delivering notes and examples to students. Students discover the mathematics themselves and basically write their own notes down about the topic after their discovery. For example, in Investigation 4 of the book “How Likely Is It?”, students tackle the ideas of experimental and theoretical probability. The students actually perform different probability experiments by drawing different colored blocks out of a bucket. Then students are asked to make connections between the experimental and the theoretical probabilities. Problem 4.2, letter E asks, “Compare the theoretical probabilities you found in part C to the experimental probabilities you found in part C. Are the probabilities for each color close? Are they the same? If not, why not?” In more traditional curriculums, students are asked only to find the theoretical probability. This curriculum has the students find both, and then asks them to communicate the difference. They can confer with other students to do this and by analyzing their data. Then, each student must communicate their findings in their journals, where they keep their investigations. This activity, as with most in Connected Math, is a great example of how students learn more by being active participants in mathematics, in many different ways, especially writing.

The next major component of the communication standard is that students be given the opportunity to “reflect on and clarify their own thinking about mathematical ideas and situations.” This opportunity is given to students in a section called “Mathematical Reflections” which appears at the end of each Investigation in every book. This section ties together all of the major mathematical ideas from that chapter. It usually consists of 3-5 questions. For example some questions from the Mathematical Reflections from the above mentioned probability investigation are as follows, “How can you find the experimental probability of an event? Why is this called an experimental probability? How can you find the theoretical probability of an

event? Why is this called a theoretical probability? Think of some situations in which it would be easier to find theoretical probabilities than experimental probabilities. Think of some situations in which it would be easier to find experimental probabilities. Explain your reasoning.” At this point, the students have already completed the investigations. These questions give them the opportunity to reflect on what they have learned. In inviting the students to think of specific instances where certain probabilities would be easier to find, this questions allows them to demonstrate creativity and to make connections to situations in real life. Another very important quality of this component of Connected Math is that this section can be used as an alternate way of assessing student knowledge other than tests and quizzes. Some teachers in my department have students work on these questions individually. They are able to use their journals in which they have completed their investigations to complete the questions. When the teachers review their answers, they are able to easily see any lapse in understanding by that student and can dialog with the student on that paper. Other teachers use the Mathematical Reflections as an opportunity for even more cooperative learning. This section lends itself nicely to a “numbered heads” situation. When using this technique, the teacher assigns each person in the group one of the questions, and that person is responsible for completing it and then explaining it to the rest of the group. As a result, any group member called upon randomly should be able to answer any of the questions. Although this technique does not involve every student writing the answer to every question, it does involve crucial dialog between students and builds communication skills.

The next component of the communication standard is that students need to “develop common understandings of mathematical ideas, including the role of definitions.” In the Connected Math teacher manuals, a list of vocabulary words is provided. Interestingly, the list is

broken into two parts, essential and non-essential. The essential vocabulary words are those that are essential for understanding in that particular unit as well as in future units. This allows the teacher flexibility to assign all or only the most important vocabulary words. Exactly how to incorporate vocabulary is also left to the teacher's discretion. I have a portion of the student binder specifically designated for vocabulary. The students record the important vocabulary words and their definitions in this section after each problem of an investigation. Some teachers in my department actually provide students with a typed copy of the vocabulary words to be kept in their binders.

Students should “use the skills of reading, listening, and viewing to interpret and evaluate mathematical ideas.” This is another component of the communication standard. Connected Math relies heavily on the skill of reading. In the beginning of each investigation, a few paragraphs are provided that actually set up the scenario for the problem to follow. Students must actively participate in the reading of these important parts of the investigation and listen as their classmates, and possibly teacher, clarify the situation before moving on to the problem. The students need to understand the basis for the problem before starting the actual investigation. This contrasts drastically with traditional programs. In those programs, students learn an algorithm or set of rules to solve some mathematical concept and then they might use that algorithm in solving a word problem later, usually at the end of the chapter or at end of a homework assignment. It makes much more sense to first develop a real life situation that will use the mathematics that the student is about to explore. The students definitely find much more value in this type of mathematics. This is the first year in my eight years of teaching that I have not heard the question, “When are we ever going to use this?”. The only problem with this component of the program is that because it relies heavily on reading, teachers must help those

students who are not strong readers compensate. Many teachers do this by having certain students paraphrase after reading a section to clarify the situation for the class. Others simply quickly conference with those students who are weaker in reading while the others are getting started. This can be done discreetly as the teacher usually is circulating around the room during an investigation anyway. The special education teachers in my department have dealt with this issue while using Connected Math by slowing down and reading most activities and questions out loud. When doing this, they not only concentrate on the mathematics vocabulary, but any word which may be giving their students a hard time.

Another key aspect of the communication standard is that students should be given opportunities to “discuss mathematical ideas and make conjectures and convincing arguments.” The Connected Math Program centers around this idea. As an educator, it has been fabulous to listen to students’ ideas and conjectures during this past school year. If students have varying answers, they want to share them. Because they have completed experiments and investigated the problems in depth, they feel confident that their response is correct, although it may be different than another student’s answer. They now know that there is not always just one right answer in math. Because they are asked to explain their thinking on a regular basis, they know how to argue their point convincingly, both on paper and in a discussion. Furthermore, since the problems in the investigations do not come with algorithms for the students to use to solve them, students are forced to make conjectures to solve problems. Sometimes their first way is not the best way. Sometimes different students will come up with different conjectures both of which are absolutely correct. It is this component of the program that allows students to think in the way that best suits them, whether that be numerically, pictorially, or some other efficient method.

The last component of the mathematics as communication standard is that students should be given opportunities to communicate so that they can “appreciate the value of mathematical notation and its role in the development of mathematical ideas.” When formulas and algorithms are used arbitrarily and out of context, students tend to just memorize and not focus on the necessity of the mathematics. Through first discovering the concepts, often involving lengthy representations, students realize just how important math symbols are. For example, in Bits and Pieces I, students are asked to represent percents and decimals using hundredths grids. Students are given a list of 100 cats and their different physical qualities such as eye color and gender. Students may be asked to determine what fraction of the cats have green eyes. They may shade in 58 out of 100 blocks if they count 58 of the cats have green eyes. They later make the important connection that instead of shading in 58 blocks, they could just state 58 out of 100 or 58% of the cats have green eyes. They realize that shading a hundredths grid for every problem like this would be too time consuming. However, they needed the hundredths grid to help them understand what percent actually means. This helped them transition to the symbolic form of a percent sign.

Most of the above mentioned components of the “mathematics as communication” standard share one common strand: writing. While teaching with Connected Math over the past school year, I have discovered how well it focuses on writing in the mathematics classroom as a means of communication. Over the years, I tried to incorporate this skill into the more traditional curriculums from which I have taught. I found this task to be tremendously difficult, with this year being the exception. I would use journal entries, math logs, open-ended assessments, and the like, but my efforts seemed scattered. I never felt that I incorporated enough writing exercises to bring them all together. With Connected Math, every day involves

journal writing. Every day involves vocabulary. Each lesson is basically an open-ended problem that the students are asked to explore and investigate.

Proponents of traditional curriculums may question the importance of all of the conjecture, exploration, and writing in Connected Math. Traditional programs focus more on algorithms and mastering techniques. Reform curriculums, such as Connected Math, focus on connecting mathematics to the world around us. The crucial piece of this puzzle is writing. It is the written story or scenario in the beginning of each problem of an investigation that gets the students interested in the mathematics they are about to explore. It is the student's writing about what they have discovered that allows them to then make the jump to the symbolic mathematics. For example, in Problem 3.6 of "Shapes and Designs", after already covering angles and angle measurements, students read about Amelia Earhart's last flight and its tragic outcome. Then students measure angles involved with the flight to find out how many degrees off course Earhart's crash site was from her destination. They find out how a small measurement error, in this case eight degrees, can cause disastrous results. A traditional program would have students measure angles in independent problems with very little meaningful application. This historically slanted problem shows that math is of vital importance.

The importance of actually writing cohesive statements in solving and reflecting upon the type of problems in Connected Math is invaluable. Joan Countryman stated this importance in her book, "Writing to Learn Mathematics":

"Knowing mathematics is doing mathematics. We need to create situations where students can be active, creative, and responsive to the physical world. I believe that to learn mathematics, students must construct it for themselves. They can only do that by exploring, justifying, representing,

discussing, using, describing, investigating, predicting, in short by being active in the world. Writing is an ideal activity for such processes.”

This is the basic philosophy of the Connected Mathematics Project. Students do all of those things when engaging in mathematics. The importance of writing as a vehicle to do these things is vital. Ms. Countryman also summarizes why this is so important:

- “To increase confidence
- To increase participation
- To decentralize authority
- To encourage independence
- To replace quizzes and tests of a means of assessment
- To monitor progress
- To enhance communication between teacher and student
- To record growth”

I have found that Connected Math has done all of these things, although some of them not very easily.

Decentralizing authority has been an issue for teachers in my department. It is difficult for many teachers after being the focus of lessons for years and years to simply turn their classrooms into one that is totally student centered. Although most are getting better at it as time goes on, some still want to try and take a constructivist lesson and turn it into a lesson where students take some notes and then complete an activity. To combat this, my district mandated from the beginning of the year, that teachers may not supplement the program with any other worksheets without approval first. Although this takes an aggressive and sometimes unpopular stand on the issue, many feel that for the program to be a success, it has to be implemented in its

true, original form, at least the first time through. I still have noticed teachers trying to put their own slants on lessons as this type of teaching is so different from what they have done before. Some do not trust that the program will make the “connections” it guarantees. I believe this fear and anxiety will diminish with further inservicing as the teachers understand what the 7th and 8th grade curriculums entail and they realize how it does connect with the 6th grade program. Personally, I have enjoyed the change as since the beginning of my career, I have tried to implement similar activities into the traditional curriculums, although my efforts were sporadic. Connected Math incorporates similar activities to what I tried to do, however on a daily basis.

Increased participation, confidence, and independence are all extremely common characteristics of students engaging in this program. It has been a joy to read the answers of students who are too shy to speak in front of the class or to read the answer of student who is not very good at algorithmic math, but given the opportunity to solve in a different way, can complete the problem. Opportunities for this type of assessment of student knowledge are not just in the investigations and mathematical reflections sections. They are also in the homework and on written assessments included in the program. Students are constantly being asked to solve problems and to explore, investigate, and explain. I have found that students have gotten very comfortable explaining themselves. They know that they can earn partial credit with a good, mathematical rationale, even if their answers are not perfect.

Connected Math has successfully incorporated writing in meaningful ways into the mathematics curriculum as well as many other modes of communication. I have immensely enjoyed using this program for these reasons as well as for many others, as it focuses on so many important areas of math education. I look forward to learning even more about the program as

our district inservices progress and we meet other professionals who are involved with the program.

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