COLLECTIVE LEARNING IN EVERYDAY MATHEMATICS CLASSROOMS

EDC 566A FINAL PROJECT : CORE+ MATH 3

Mark H. Hoffman
Liberty High School
1115 Linden Street
Bethlehem, PA. 18018
(610) 69107200 ext 7278
May 1, 2002

Introduction: Need for Educational Reform

Nearly all mathematics educators have, by now, read or heard about reports showing United States students doing poorly in mathematics (SIMS, 1982). This has naturally sent out a call for reform, which has resulted in various levels of disagreement as to the diagnosis and prescription. Some researchers pin their hopes for improvement on direct instruction and more carefully supervised practice. Others recommend greater use of small group arrangements in which students help each other, gain incentives through competition with other groups, or learn through engaging in mathematical activities. Many people, other than mathematicians, engage in mathematical activities, with interests varying over a wide range. But while doing so, we all have to follow the same basic steps to get results: hypothesize, experiment, execute mathematical procedures, communicate and defend our conclusions. These activities not only represent what it means to be a good mathematician, but also to be a good scientist, a critical thinker, and a problem solver. This means that learning mathematics requires construction of concepts based on prior knowledge, not simply passive reception of facts.

Researchers have shown that teaching can only be effective if it causes students to respond to and link their own prior knowledge, to new discovered knowledge. For students to have a stake in their learning, they need to explore mathematics on their own, constructing strategies to solve problems. For example, in solving a quadratic equation, instead of teaching the quadratic formula and assigning a dozen problems to reinforce the skill of using the formula, allow the students to explore other approaches to the problem: estimating, factoring, graphs, using calculators which produce graphs and tables. Each student then should be encouraged to express his(her) approach both orally and in writing, and argue convincingly for his(her) approach. This should be accomplished amid and arena of conflicting ideas and strategies. Learning mathematics, therefore, is now becoming a shift from just accumulating facts and procedures, to learning how to use an integrated set of intellectual tools for making sense of mathematical problems, and using problem solving skills (NCTM, 1991), (Heaton, 2000). This requires working with mathematical objects in a mathematical community. Therefore, teachers need to find a way to establish a *mathematical environment* in their classrooms.

In developing a new method, then, for student learning of mathematics, there first has to be a clear understanding of the long-term objectives of mathematics teaching. First and foremost, I believe it is objective to teach students how to be mathematical problem solvers, and learn how to reason mathematically. Once students have mastered these skills, they will become more confident in their own abilities to apply mathematical knowledge to new problem situations in their own world. In addition, to be effective problem solvers, students also need to be able to communicate their results in clear, succinct mathematical terms. Teaching problem solving successfully involves teachers thinking about mathematics as more conceptual than rule governed, emphasizing working with real world applications problems. These problems need to be of a generally higher degree of difficulty; getting the students involved with more difficult subject matter. Westbury (1994) refers to this type of teaching as *Cognitively Demanding Instruction.*

Therefore, when designing any new method for teaching mathematics, and the curricula that it supports, it surely must support these objectives. In addition, the curriculum must be intrinsically motivating and challenging, engaging the students in reading and writing. The curriculum needs to stress mathematical concepts, causing the students to use reasoning, visual / spatial acuity and computational skills, and require the students to be precise in their work. Conceptual learning

requires tasks that include hypothesizing, estimating, experimenting with manipulatives, with consideration of open-ended questions.

Background and Rationale for Cooperative Learning

Research and educational reformers of mathematics education have been critical of the use of lecture centered teaching as the means for student learning. Many of these critics argue in favor of alternative teaching methods, arguing that if learning occurs at all via lecture, it is passive and tends toward memorization. In my experience, many hand mathematical tools and or short-cuts to algebraic procedures can really only be useful if they are instantly available from recall, for use to simplify algebraic expressions. Therefore, to me, memorization of certain identities or formulas may not be a bad thing. However, the voice of research has suggested alternative styles of teaching. One popular method is some form of small group activity. Often the small group instruction is labeled as cooperative learning. Unfortunately, working in small groups does not guarantee that any learning or cooperation is even taking place. However, as calculators and computers in the classroom become more available, mathematics educators at all levels began experimenting with ways to enhance student understanding of mathematical concepts. As students worked with these tools, educators observed an increased interaction among the students. Some instructors noticed that students were remembering more and apparently understanding important concepts more deeply. These educators soon began to exploit this style of learning, informally at first, then in more structured ways. Finally, the use of cooperative learning became an integral component of their courses. There does not seem to be one clear cut definition of cooperative learning, as researchers seem to have rather diverse opinions about what cooperative learning means. These range from loosely organized to highly structured classroom settings.

We all remember traditional classrooms, where students were expected to sit quietly and listen attentively, assuming a rather passive role in the instructional process. The teacher had to, from time to time, exert strong control to keep students quiet and on task. This took time away from instruction and learning, especially in a double block environment. The idea of a group centered learning environment is to channel student energy by actively engaging them in the learning process. This, of course, assumes that peer group interactions are constructive forces that enhance academic learning. In trying to apply this idea to mathematics, it is the hopes that the group would provide both social and technical support to its members, in solving problems collaboratively, promoting a sense of accomplishment. It would alleviate the problem with students sitting alone, being stuck on a problem, and give all the members of the group the opportunity to share in the success of solving the problem. It is thought that the group interactions help all members learn the concepts and problem solving strategies.

Group learning ties together three popular theories of learning that have developed over the past thirty years: Piaget's *Theory of Cognitive Development*, (Piaget, 1970), *Constructivist Theory of Learning*, and metacognition. Much has been written in these areas, and it is not the goal of this paper to discuss these subjects in depth. Rather, I want to simply tie these together as the genesis of collective learning theories. According to C*onstructivist Theory of Learning* (Davis, et al, 1992), the learner constructs new knowledge by an adaptive process of organizing and making sense of new experiences by making bridges to prior knowledge or experiences. In Piaget's view of learning, individuals construct knowledge by interacting with their environment. This includes, to a large extent, social interactions. Finally, by interacting together, students in a group environment can discuss various strategies for solving problems. This thinking about thinking, or *metacognition*, becomes a sort of self-managed learning within the group. However, students need to be able to come into a group setting with self-management skills; being able to

successfully socially interact with others in a group. There is, an active interplay between the construction of mathematical concepts and the development of cooperative, social skills.

When considering the merits of group discussions, mathematics problems are well suited for this type of learning environment. Most mathematics problems can be solved in a reasonable length of time. In addition, logical arguments can be used to demonstrate how to find the solution. Discussion is focused and stays objective. Different approaches to a problem can be discussed and the merits of each graded by the group. Learning takes place when students have the ability to communicate concepts to others. The bottom line here is to find a way to motivate students to learn mathematical skills and concepts and to encourage and help each other achieve mastery. The National Council of Teachers of Mathematics (NCTM, 1991) states the needs for the following shifts in teaching and learning mathematics as follows:

1. toward classrooms as mathematical communities and away from classrooms as simply a collection of individuals
2. toward logic and mathematical evidence as verification and away from the teacher as the sole authority for the right answers.
3. toward mathematical reasoning and away from merely memorizing procedures.
4. toward hypothesizing and problem solving and away from simple mechanical answer finding
5. toward connecting mathematics to real life applications and away from treating mathematics as a body of isolated concepts and procedures.

The NCTM further imagines the teacher as the *facilitator* of learning rather than the imparter of information. This is rather like a musical conductor, someone who plays a central role in orchestrating the oral and written discourse in ways that contribute to students' understanding of mathematics. The motivation then, in educational reform, is to create a community of learners, who actively work together in small groups, to enhance each member's mathematical knowledge and proficiency, in a positive, enjoyable work environment.

Definition and Goals of Cooperative Learning in Mathematics

The most general and all encompassing definition of cooperative learning is simply a diverse group of instructional methods in which small groups work together and aid each other in completing academic tasks. The goal is to provide an environment where student learning is supported, by giving them opportunities to describe their own ideas, hear others explain their thoughts, and explore various strategies for solving problems. Hagelgans (et al, 1995), identifies several characteristics that must be evident for the groups to be called "cooperative learning groups". These groups must be stable, permanent groups in which a significant amount of work in the course is accomplished. Within these groups, a positive work ethic and camaraderie, an esprit *de corps* exists. Team members share responsibility for accomplishing the tasks assigned and for each other's learning. The results of the group work are included in the evaluation process of each individual student's grade.

In a traditional classroom, the teacher is the key person in the learning situation, providing information, explaining concepts or skills, and giving examples. Instructional flow is generally one way, with the teacher directing the instruction, leading the lessons, prompting responses, and paces the class. Teaching in more of a collaborative / cooperative environment, requires a different role of the teacher. Rather than direct a lesson, the teacher needs to provide time for the students to discuss the problem, search for methods to solve it, and learn how to evaluate the solution. In a collaborative classroom, the instructional flow follows more of a cycle of activities, beginning with the teacher presenting an overview of the lesson, goals, and a short lecture or whole-class instruction. The class is then divided into groups to accomplish their assigned tasks. After the group activity is completed, the class is brought back together to synthesize and summarize the concepts learned. The group activity is structured, following some simple, basic rules. These rules assure mutual cooperation and helpfulness, by requiring each student to have equal status. Each student has a role, to assure individual accountability. These roles can vary, and may be mandated by the curriculum. One example of assigned roles are: group leader, recorder (secretary), and reader (Coxford, et al, 1997).

The most prominent methods of cooperative learning have been developed by scholars and researchers alike (Slavin, 1991), (Johnson & Johnson, 1989), and (Cohen, 1994). In the past ten years, there has been a considerable body of research associated with the development of cooperative learning methods. However, these methods are largely experimental, having been conducted in laboratory or field experiments, under controlled conditions. In these studies, cooperative learning has been shown to be effective in raising academic achievement, self-esteem, and in improving students' attitudes toward school and their peers. Most researchers agree that collective learning classrooms and groups should begin by being multi-dimensionally heterogeneous. By this is meant that the classroom should be mixed by academic ability, language diversity, ethnical/social/cultural backgrounds, and by emotional/social maturity.

The roles of teachers and students, and the dynamics of the group, follow certain guidelines, which will be discussed in the next section. However, the primary focus of cooperative learning, is on the students' own thinking processes. In order for this to occur, the students must already have mastered various problem-solving strategies, have confidence in their abilities and be willing to risk being wrong. Once the students have an answer to the problem, they must have the ability to evaluate the answer (ie: does it make sense). These are skills that do not necessarily come naturally to the students, and may have to be initially taught to assure that the groups have

the necessary social skills to be successful. As opposed to the traditional classroom setting, in a cooperative learning classroom, students speak more often and become comfortable with taking risks of trying out their thinking during problem-solving situations.

Classroom Dynamics: Teacher and Student Roles

The simplest type of team learning takes the form of four or five member heterogeneous teams. In this environment, the teacher introduces new material in a lecture or discussion. Team members then study worksheets on the material, or work problems, as a group. The teams are tasked with not being finished until each member of the team understands the material. Following team practice, students take quizzes on the material. Teammates may not help each other on the quizzes; they are on their own. The individual teammate scores are formed into a team score. Team scores are compared with previous scores, with bonus points awarded if they exceed prior performances. Teams are encouraged to compete. The team earning the highest number of points earns a certificate. This encourages team members to help each other and even study together out side of class. Therefore, it is the main goal of the team to prepare its members to do well on quizzes. The whole of this activity is to foster team competition, like in team sports. Slavin (1991) uses the metaphor of a baseball team, characterizing the teacher more as a *coach,* than as a *boss*. The winning team can have its own "MVP" for that given week, and students with perfect scores can be showcased. Learning then becomes a social activity instead of isolated exercises, fun instead of boring, yet under the groups' own control instead of the teacher's. Delegation of authority is required with independent groups working, because the teacher cannot be everywhere at once, managing student learning and behavior.

The teacher's role in these activities is one of a leader and a catalyst, which helps students think mathematically. The teacher is really an intellectual coach who is a role model to the students. The teacher leads, demonstrates, and sets the standard for high-order thinking. The teacher is a consultant, a moderator, and a participant in discussions. Once a problem has been introduced, the students can work collectively toward a solution. It is the teacher's responsibility to encourage student interactions, to ask questions to stimulate discussions with the groups, and to encourage students to hypothesize about the problem situation. The teacher needs to encourage students to work together using the norms and rules established in the classroom. This can be accomplished while the students are engaged in exploring the solution to the problem, observing the various groups' strategies and offering assistance where needed to get / keep the groups on task. It is important to be aware of groups that finish early, to provide some extension activity until the other groups catch up. When a group is stuck on a problem, the teacher should help the group restate the problem in their own terms, re-explaining concepts and posing simple examples. If the group is pursuing a completely wrong line of reasoning, then the teacher needs to point out errors, while not giving them the correct answers. A good rule for groups is that students may ask the teacher for help only when everyone in the group has the same question. After the groups have completed their assignments, the teacher should have them restate their solutions, showing their procedures and strategies used, in an organized presentation, using various forms of multi-media. Once all groups have displayed their solutions, the teacher can lead a discussion on generalizing the solution and extending it to other situations. One method is to generate a mathematical model which can be used to predict future trends. The limitations of these models need to be discussed as well as how they can be applied in similar situations.

The primary job of the group is to see that each member of the team has reached the learning goals. Thus, students have to seek outcomes that are beneficial to all those in the group. It is the responsibility of the students themselves, to see that the members of the group follow the group norms and rules established in the classroom. Students need to encourage each other to try to understand the solutions and provide help to each other as needed until all students in the group

have achieved mastery of the material. Though students have heard much about cooperation, functioning cooperatively is not a skill with which they necessarily have much practice. Meeting the responsibility for their own work and behavior within the group does not come naturally. Both interpersonal and small group skills must be practiced. Learning groups are not productive unless its members are skilled in, and are motivated to, cooperating with each other. These skills include leadership, decision-making, trust building, communication, and conflict management skills (Johnson & Johnson, 1989). These skills have to be taught early, just as purposely and precisely as teaching mathematical skills. Students must have a cooperative attitude about work, a commitment to their peers, setting all prejudices aside.

A teacher interested in increasing the students' cooperation and/or cooperative attitude can begin in small increments. This may be particularly difficult with older (middle and high school) students who are not used to working in a cooperative situation, having been independent for many years, or are less than enthusiastic about mathematics. A simple rule then should apply when students are working in-groups: no one is allowed to interfere with another students' learning (Davidson, 1990). This rule is rather all encompassing, eliminating the need for rules about no hitting or no whistling out loud, as those acts would certainly interfere with student learning. This also means that everyone's opinion has the same weight. To facilitate discussions and minimize confusion, only one person talks at a time. There is no name calling, and no one should use derogatory remarks or say "shut-up".

Problems Encountered in Everyday Classrooms

Though cooperative learning has its benefits, it also has its share of problems, especially if the students have little or no experience with working collaboratively in small groups. The first problem traditional-teachers face is the increased noise level during group work, and the apparent confusion during transitions from one activity to another. Other very real questions arise, especially when trying to involve low achievers in-group work. These questions include how to deal with students who are disliked, with students who refuse to work in a group, students who are not motivated to do mathematics, or with ethnically diverse students, ESOL students, with wide varying reading comprehension and communication levels. In these cases, instead of being a facilitator, the teacher needs to present or review the concepts needed, pose the problem situation and make the students aware of what they are supposed to do.

The simple installation of cooperative learning methods is not sufficient in itself, to deal with low-performer problems. To the contrary, time must be spent teaching students how to work in a collaborative environment. The primary solution is time. Some students will always be initially unhappy about their team assignments. The key is to convince then that the team needs to cooperate to be successful. The proof will be in their first team scores. That may be enough of a motivator to make them get along. Some students, however, will need constant reminding that their task consists of as much cooperation with their teammates, as solving a mathematical problem. On some occasions, students may simply refuse to work with a team. In these (hopefully rare) cases, these students should be allowed to work alone until they are ready to work as a team. The Core-Plus integrated mathematics curricula (Coxford, et al, 1997), assumes all incoming students are accustomed to working in groups, and meet all academic standards for eighth grade level mathematics. Many low-performer students, with low reading levels, have trouble understanding the presentation of situational problems. This requires the teacher to spend more time on helping students rephrasing the problem in their own words, and providing guidance on how to tackle the problem.

There has, to a large degree, be a match between the instructional goals and the organization of the classroom / teaching environment. Students, who are struggling with basic mathematical skills, may first require more practice or mechanical memorization of procedures, needed to solve higher order problems. Low-level students who are struggling to master mathematics skills cannot be expected to perform well at solving conceptual problems. These students require the opportunity to develop these skills first before learning problem-solving strategies. It is like trying to repair a car with an empty toolbox, or with simply a screwdriver and a hammer. Skill development still requires pencil and paper tasks, or appropriate workbook assignments. These too, can be handled within a group setting.

A greater problem than with the mathematics, is getting a group to focus on what they are supposed to be doing and how they need to organize to solve the problem. To this end, the teacher needs to provide clear expectations or goals, the appropriate materials, time limits, and the methods for reporting their results. Nevertheless, discipline continues to be a major obstacle for many teachers. For many reasons, students today are not as obedient, respectful, polite and placid as they were in previous generations. To help assure cooperation between team members, the teacher needs to set a firm tone on the first day that cooperation is required. Teachers should be emphatic that putdowns, making fun of teammates, or refusing to help other members of the team are not acceptable kinds of behavior. Though counter to the idea that students should be

self-motivated, researchers (Slavin, 1991), encourages teachers to implement a reward system (extra team points) based on the team's behavior and cooperativeness and effort. However, it is the teacher's responsibility to inform the teams when they are doing well, and to set the standard for acceptable behavior.

Aside from the discipline problems, group learning may not be working for the following various reasons:

1. A proper curriculum is not available. The choice of text and materials are crucial because a conventional text does not often work in small group learning circles. These are texts with repetitive drill exercises witch reinforce mechanical skills. These do not allow a student to learn how to think through a process. On the other hand, Core-Plus (Coxford, et al, 1997) integrated curricula provides so few examples that it is difficult for the students to discover the concepts being taught.
2. Small groups may become bored with the curricula or each other and either stop talking to each other, or talk about other topics unrelated to mathematics. In this case, the group needs a strong leader to insure that the group remains focused and motivated.
3. Students may feel defeated by a difficult exercise because of poor reading skills and not understand how the problem is defined or what is required. If the students are all struggling with an idea, then the teacher should lecture briefly, as necessary then let the groups again discuss the problem.

As a consequence of group interaction, classroom noise will naturally increase. This causes a lot of consternation on the part of teachers, who are used to students working quietly, as a sign of being on task. By the same token, team learning does not go well when the teacher is asking the students to quiet down every five minutes. But if things begin to get out of hand, something needs to be done to regain control. The first solution is to bring the activity to a halt, obtaining absolute quiet, then whisper to the students to speak softly. Students should be taught to stop talking immediately when the lights are flicked off for a moment or a chime is sounded, or other audio/visual signal is given. If none of this works, then make the noise level part of the criteria for earning extra team points. Of last resort, terminate the activity and revert to a whole class discussion or individual work for the remainder of the period.

In Support of the Teacher: Curricular Needs and Staff Development Requirements

Interest in mathematical topics and activities is intended to be a major source for student motivation. This places a large burden on the curriculum to provide problems that arise in real life; real life situations such as motion, radio active decay, profit and loss, and financial growth. In these situations, the goal is for the students to discover a pattern in the data and the relationships between the variables. The authors of curricula perceive these problems to be interesting, valuable, and of practical use to the students. In all practicality, the goal of providing an interesting, learning environment is not easy to achieve. In mathematics, the subject matter proceeds from the concrete to the more abstract, theoretical considerations. Emphasis is placed on discovery of new ideas based on students' previous knowledge. Therefore, the teacher has to rely on the students' own internal sources of motivation, such as curiosity, and interest in mathematical topics.

In the overwhelming majority of classrooms, the content of the textbook governs what is taught and how it is taught. Teachers may edit or delete sections of text, but seldom will they teach topics not covered in the texts. Neither will most teachers approach a topic differently from the way it is treated in the text. Therefore, the curriculum determines which principles of mathematics are taught as well as the strategies for learning. The textbook must reflect the concepts of mathematical education. That is, reflect the principles of group centered education, for cooperative learning to be effective. Unless the text contains engaging projects and motivating group activities, few teachers will have the time to create their own. Unless textbooks include suitable assignments to enhance students' experience with reading and writing in the language of mathematics, students will remain mathematically deficient (Reshaping School Mathematics, MSEB, 1990), (Cohen, 1994). Therefore, curriculum plays a large role in the success of cooperative learning, needing to be specifically designed for group study.

It is clear that the classroom management system of a cooperative learning environment include activity-based learning to keep the students engaged. This requires selected supervision, by teachers, when required, to make sure the tasks are being completed and that the students are engaged in learning. This requires considerable training of both students and teachers, to design and accept classroom norms and rules for delegation of authority to succeed within the groups. To assure success of such a demanding program, teachers require support from the building principal to include adequate classroom facilities and resources, adequate prep time and training to develop and implement complex instructional plans, and manageable (small) class sizes. Cooperative learning has shown a strong record of success in experimental settings, but it has not always fared well in studies in *everyday classrooms*, especially when teachers have not received strong external support from administrators and school board (Jacobs, 1999).

The term *cooperative learning* has so many interpretations, models, and has generated many debates about philosophy, research, and implementation, over the past thirty years. Teachers attempting to implement cooperative learning methods have to make informed decisions about selecting and evaluating many cooperative learning approaches. This takes into consideration diversity in school context, students, and curricula. Therefore, professional development of teachers has become even more important as issue. This requires regular and sustained support from administrators in terms of in-service training to equip teachers with new classroom

strategies and experiences to assist and improve the achievement of their students. (Brody, 1998). This can come from instruction on best practices by consultants, teacher educators, and staff developers (Zemelman, et al, 1993). The responsibility for continuing staff development is a serious one, and every bit as important as the cooperative learning strategies themselves. Otherwise, teachers will cling to what they are familiar with, and not stray far from the limits of their existing experience, curricula and textbooks.

Synthesis of Research Data and Conclusions

Much of the research conducted in the 1980's was centered in mid-Atlantic, rural, suburban communities. In these communities, there was much evidence to support the success of cooperative learning. Ironically, researchers have also found that urbanization is accompanied by a decrease in the kind of *natural* cooperation found in the rural areas and traditional societies. Traditionally, neighbors helping neighbors was essential for the survival and growth of rural / farm communities, where goods and services were at a minimum. In contrast, today, urban lives take on an anonymity, to such an extent that neighbors are now in individual competition with each other (Brandt, 1991). This means that the ability to work constructively with others, which at one time was passed on by parents and other elder family members, must now be formally taught in schools, for cooperative learning to work. While this trend of urbanization and resulting in breakdown of family values is unavoidable, it makes implementation of group learning all the more difficult. Therefore, expecting cooperation in the classroom runs counter to student and most adult present day values. They expect education to be a fiercely competitive enterprise. This is evidenced by the high emphasis on high GPA and SAT scores for college admission. Overcoming these paradigms is required for success in a cooperative learning environment. Independence, individual initiative, and personal achievement, though important values need to be balanced with a new emphasis on sharing, consideration, and commitment to group, rather than to individual goals.

It is clear that cooperative learning is not students sitting together in-groups, working on problems individually, or letting one person do all the work. Admittedly, this could, more often than not, take place when students are thrusted into a group learning situation without adequate training or motivation. True cooperative learning capitalizes on the presence of student peers, encouraging student–to–student interaction. In essence, the students establish a symbiotic relationship among themselves. (Posamentier & Stepelman, 1995). Students in *effective groups* learn to listen to each other's ideas, discuss and disagree, offer and accept constructive criticism, and be comfortable about making mistakes. In structuring small learning groups, heterogeneity is agreed by most researchers, to be preferred. Optimum group size varies from two to five, depending on group cohesiveness and how well they function. Designing an appropriate task is as as important as group dynamics. Lastly, a well-designed reward structure gives added incentives for small group learning behavior among students.

Dubinsky (et al, 1997), in his review of papers on the effectiveness of cooperative learning, clearly shows that there is a great diversity in activities grouped together under the umbrella term *cooperative learning.* In attempting to answer the ultimate question of whether cooperative learning is better that the traditional lecture method, one has to first distinguish between the various types of group learning with their desired educational outcomes. These input and output variables of educational outcomes are extremely complex, and are beyond the scope of this paper to fully analyze. Suffice it to say there is *not* one cooperative learning strategy that is universally better than lecture-based instruction. The complexity of the problems of teaching and learning mathematics indicates that rather than being provided a panacea for learning, teachers will have to, for themselves, select the environment and teaching style that works best for them. In practical terms, this will manifest itself as a combination of methods and/or a variation or rotation of methods, depending on the particular needs of the class.

Researchers have been busy documenting the advantages of cooperative learning over traditional methods, and the differences in outcomes among cooperative learning methods. It was as if the teacher's main job was to choose among the various structures. The question should now become not which cooperative learning method to use, but rather how to combine various structures to design the most powerful lesson plan. These structures may include activities that promote networking among students, emphasize concept development, teamwork, and group projects. These activities may also help develop mastery of skills through practice exercises and allow brainstorming of ideas and strategies for problem solving (Andrini, 1991). These activities *combine* various teaching methods within the cooperative learning environment, lecture, and skills development exercises.

REFERENCES

Andrini, B. (1991). *Cooperative learning and mathematics: A multi-structural approach.* San Juan Capistrano, CA: Resources for Teachers, Inc.

Brandt, R. S. (Eds.). (1991). *Cooperative learning and the collaborative school: Readings from educational leadership.* Alexandria, VA: Association for Supervision and Curricular Development.

Brody, C. M., & Davidson, N. (Eds.). (1998). *Professional development for cooperative learning: Issues and approaches.* Albany, NY: State University of New York Press.

Cohen, E. (1994). *Designing group work: Strategies for the heterogeneous classroom.* Albany, NY: Teachers College Press.

Coxford, A. F., Fey, J. T., Hirsch, C. R., Schoen, H. L., Burrill, G., Hart, W. E., Watkins, A. E., with Messinger, M., & Ritsema, B. (1997). *Core-Plus mathematics project, contemporary mathematics in context: A unified approach.* Chicago, Ill: Everyday Learning.

Curriculum and evaluation standards for school mathematics. (1989). Reston, VA: The National Council of Teachers of Mathematics, Inc.

Davis, R. B., Maher, C. A., & Noddings, N. (Eds.). (1992). *Constructivist views on the teaching and learning of mathematics.* Reston, VA: The National Council of teachers of Mathematics, Inc.

Davidson, E. (Ed.). (1990). *Cooperative learning in mathematics.* Menlo Park, CA: Addison-Wesley.

Dubinsky, E., Mathews, D., & Reynolds, B. E. (1997). *Readings in cooperative learning for undergraduate mathematics: MAA Notes Number 44.* Washington, DC: The Mathematical Association of America.

Dubinsky, E., & Schoenfeld, A. H. (1994). *Issues in mathematics education, volume 4: Research in collegiate mathematics education I.* Providence, RH: American Mathematics Society.

Everybody counts: A report to the nation on the future of mathematics education. (1989). Washington, DC: Mathematical Sciences Education Board, National Research Council.

Hagelhans, W. L., Reynolds, B. E., Schwingendorf, K., Vidakovic, D., Dubinsky, E., Shahin, M., & Wimbish, G. J., Jr. (1995). *A practical guide to cooperative learning in collegiate mathematics: IMAA Notes Number 37.* Washington, DC: The Mathematical Association of America.

Heaton, R. M. (2000). *Teaching mathematics to the new standards.* New York, NY: Teachers College Press.

Jacob, E. (1999). *Cooperative learning in context: An educational innovation in everyday classrooms.* Albany, NY: State University of New York Press.

Johnson, D. W., & Johnson, R. T.(1989). *Cooperation and competition: Theory and research.* Edina, MN: Interaction Book Company.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics.* Reston, VA: Autor.

Piaget, J. (1970). *Science of education and the psychology of the child.* New York, NY: Grossman Publishers.

Posamentier, A. S., & Stepelman, J. (1995). *Teaching secondary school mathematics: Techniques and enrichment units.* Englewood Cliffs, NJ: Prentice Hall.

Reshaping school mathematics: A philosophy and framework for curriculum. (1990). Washington, DC: Mathematical Sciences Education Board, National Research Council.

Second international mathematics study: International mathematics project. (1982). Urbana-Champlain: University of Illinois.

Slavin, R. E. (1991). *Student team learning: A practical guide to cooperative learning (3rd edition).* Washington, DC: National Education Association professional Library.

Westbury, I., Ethington, C. A., Sosniak, L. A., & Baker, D. P. (Eds.). (1994). *In search of a more effective mathematics education.* Norwood, NJ: Ablex.

Zemelman, S., Daniels, H., & Hyde, A. (1993). *Best practice: New standards for teaching and learning in american schools.* Portsmith, NH: Heinemann.