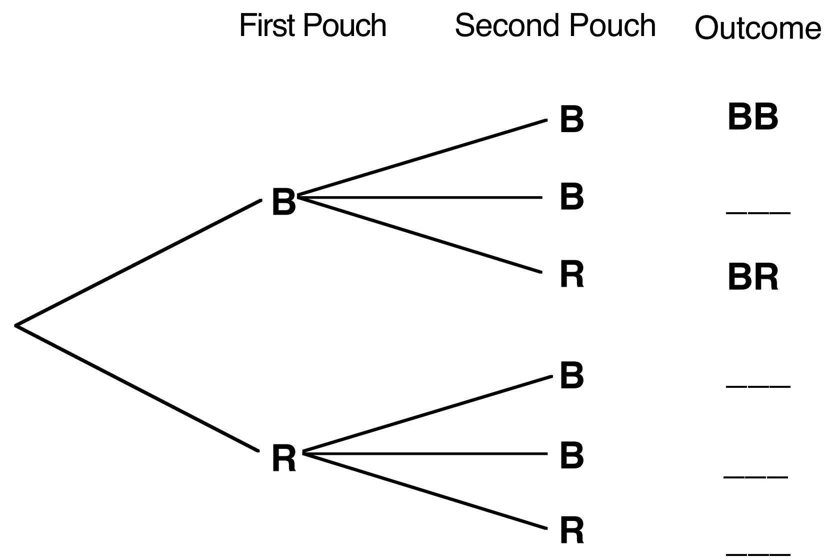
Use a tree diagram to find the number of possible outcomes.

1. A pouch contains a blue chip and a red chip. A second pouch contains two blue chips and a red chip. A chip is picked from each pouch. The chips are replaced each time. Complete the tree diagram to find the outcomes.

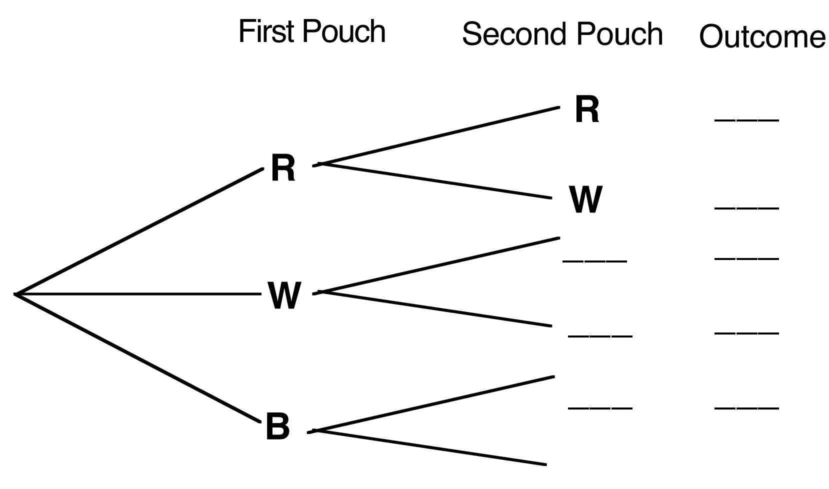



2. How many outcomes are there altogether? 2. _________

3. How many outcomes have one blue chip and one red chip? 3. _________

4. How many outcomes have no blue chips? 4. _________

5. Fill in the tree diagram below that shows the outcome drawing once from each pouch if the first pouch contains a red, a white, and a blue chip and if the second pouch contains a red chip and a white chip.



Draw a tree diagram for each problem. Then find the number of possible outcomes.

1. A choice of cereal, eggs, or French toast with a choice of orange juice or tomato juice

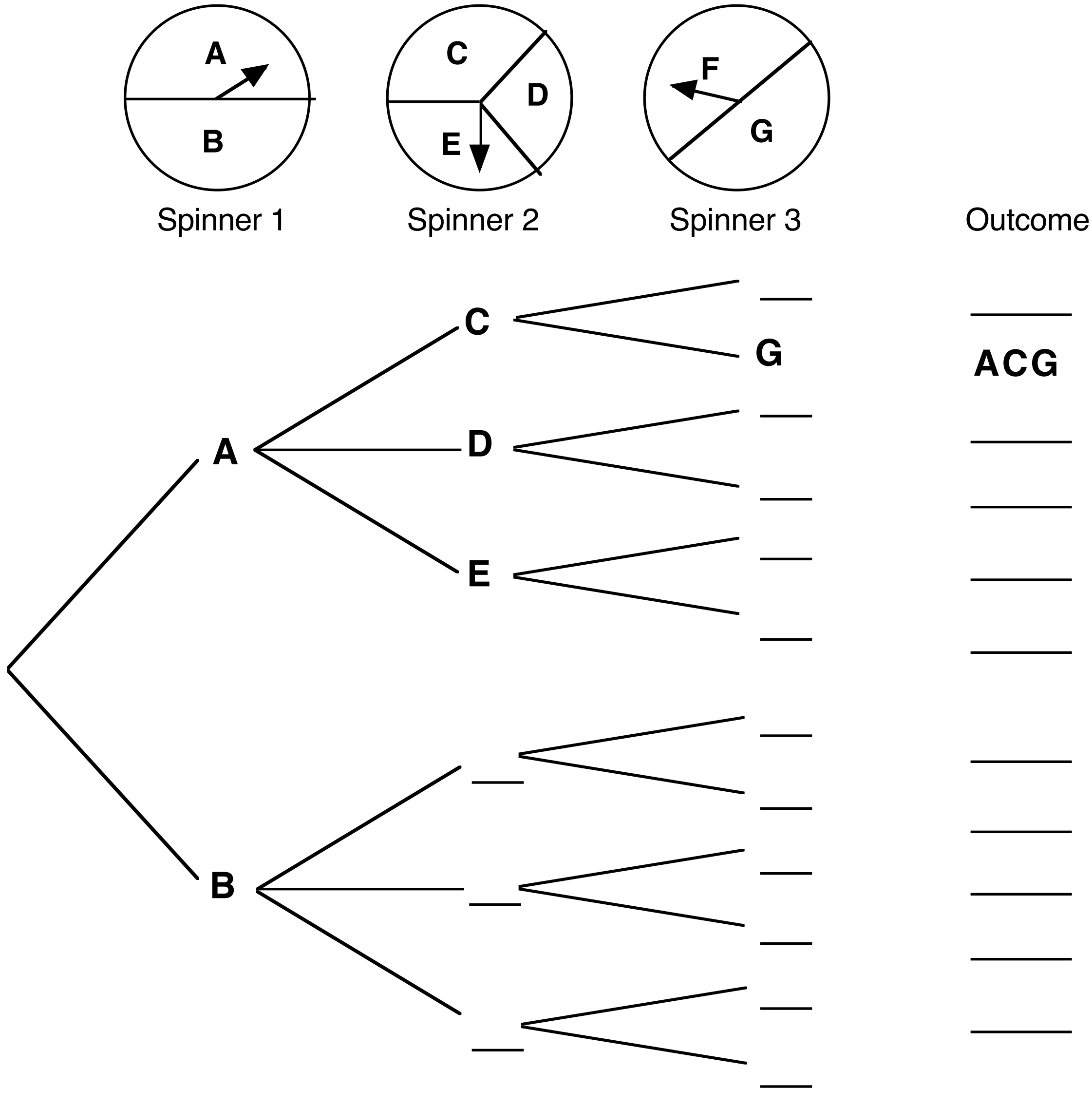
2. A choice of red, white, or blue slacks with a choice of a solid, striped, or plaid shirt

3. Flipping two coins

4. Flipping three coins

Use a tree diagram to find the number of possible outcomes.

The three spinners are each spun once. Complete the tree diagram to show all possible outcomes.




1. How many outcomes are there? _____

2. How can you find the number of outcomes using the number of possibilities on each spinner?

Find the number of possible outcomes.

1. How many outcomes are there if three coins are tossed?

2. How many outcomes are there if three dice are tossed?

3. How many outcomes are there if four coins are tossed?

4. How many outcomes are there if four dice are tossed?

5. How many outcomes are there if an ice cream store has 10 flavors and 6 toppings and you choose one of each?

6. How many outfits are there if Sue has 7 blouses and 9 skirts and 2 scarves and Sue chooses one of each?

7. How many dinner combos are there with 4 appetizers, 8 main courses, and 5 desserts if you have one of each?

8. My disguise kit contains 3 hats, 2 pair of glasses, 4 wigs, and 5 fake noses. How many possible disguises do I have if I wear one of each?

PROBABILITY WORKSHEET

Find the probability of each event.

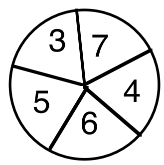
1. Rolling a die and getting a 7. 1. __________

2. Rolling a die and getting a 5 or a 6. 2. __________

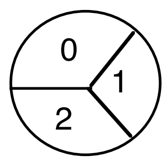
3. Picking a spade from a standard deck of cards. 3. __________

4. Picking a picture card (jack, queen, king) from a standard deck of cards. 4. __________

5. Getting higher than a 4 on the spinner shown at the right. (Assume that all divisions of the spinner are equal) 5. __________




6. Getting an even number on the spinner shown at the right. (Assume that all divisions of the spinner are equal) 6. __________




7. Picking a 4 or a heart from a deck of cards. 7. __________

8. Flipping a coin twice and getting tails twice. 8. __________

9. If you flip a coin three times, you can get any of the following results: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT.

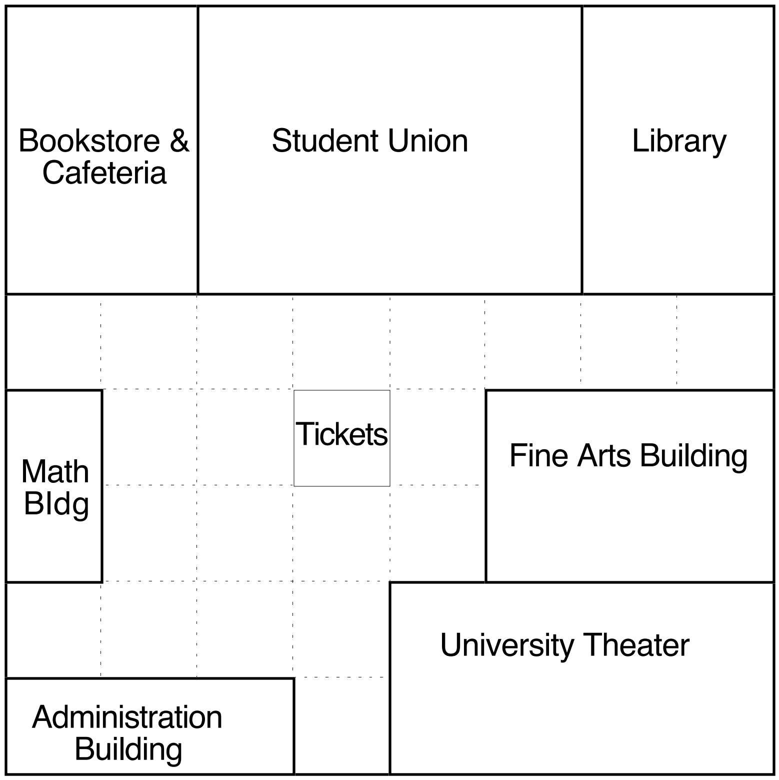
   a. Getting exactly two heads. 9a. __________

   b. Getting at least two heads 9b. __________

   c. Getting at most two heads 9c. __________

   d. Not getting two heads 9d. __________

10. Dropping a dart randomly on the rug at the right and landing on a gray area. 10. __________




A Map of part of the Student University campus is shown at the left.

Prospective students throw a dart at the map to decide where to start their tour. Assuming they are equally likely to hit any point on the map, find

P(Theater) =

P(Math Building) =

P(Admin Building) =

P(Ticket Booth) =

P(Library) =

P(Book/Cafe) =

P(open space) =

P(Union) =

P(not the Theater) =

P(not Fine Arts) =

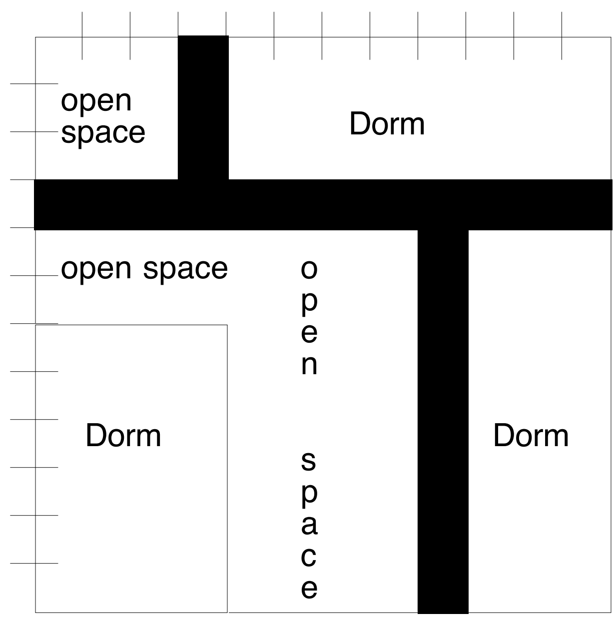
P(Fine Arts or Math) =

The map of the Student University living area is shown at the right. If a dart thrown at this map has an equal chance of landing anywhere on the map, find

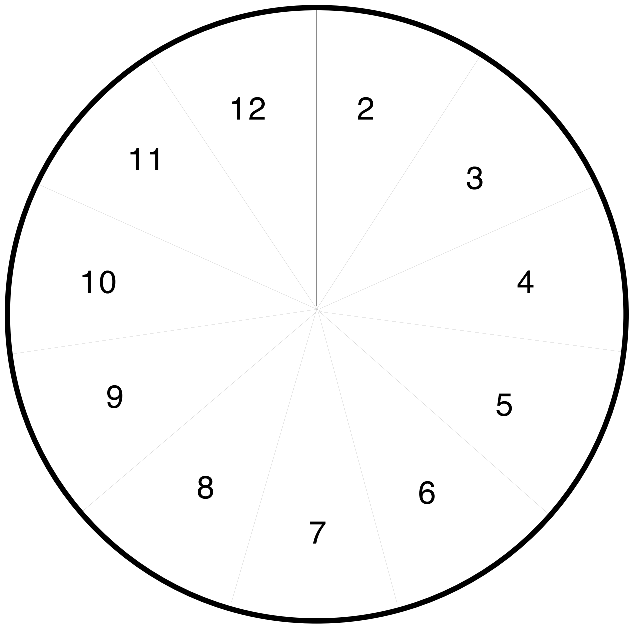
P(dorm) =

P(roadway) =

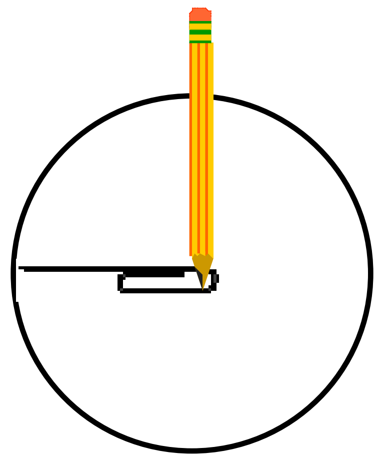
P(open space) =



The spinner below is divided into eleven equal sections. The sections are numbered from 2 to 12.




You can use this spinner by bending a paperclip for the pointer and using a pencil to hold the closed end of the clip at the center as shown below.



1. In your group, spin 50 times. Keep track of how many times each number comes up.

2. Make a table showing how many times each number comes up on the spinner for the entire class.

3. Now make a bar graph of the spinner results for your class.

4. Now roll two dice 50 times. Keep track of how many times each number from 2 to 12 comes up.

5. Make a table showing how many times each number comes up on the dice for the entire class.

6. Now make a bar graph of the dice results for your class.

7. Are your results approximately the same for the dice and the spinner? Why or why not?

This problem involves probability.

The **probability** of an **event** happening, P(event), is equal to the number of **successful outcomes** divided by the number of **possible outcomes**.

$$\text{P(event)} = \frac{\#\,of\ successful\,outcomes}{\#\,of\ possible\,outcomes}$$

For example, the probability of picking a 7 at random from a shuffled deck of cards is shown as:

$$\text{P}(7) = \frac{4}{52} = \frac{1}{13}$$

Similarly, the probability of rolling an odd number on 1 die is $\text{P(odd)} = \frac{3}{6} = \frac{1}{2}$.

**Your problem is to find the probability of rolling a 7 using two dice.**

**Answer and explain.**

Imagine a set of dice in which every 4 was replaced by a 7. So each die could roll 1, 2, 3, 5, 6, or 7, with each result equally likely.

Find the probability of each of the following results when rolling these dice.

1. The sum of the dice is 7.

2. The sum of the dice is less than 7.

3. The sum of the dice is greater than 7.

4. The product of the dice is even.

5. The product of the dice is odd.

6. Both dice are the same.

7. The sum of the dice is a multiple of 3.

8. The product of the dice is a multiple of 3.

This problem involves probability.

The **probability** of an **event** happening, P(event), is equal to the number of **successful outcomes** divided by the number of **possible outcomes**.

$$\text{P(event)} = \frac{\#\, of\ successful\, outcomes}{\#\, of\ possible\, outcomes}$$

For example, the probability of picking a heart at random from a shuffled deck of cards is shown as:

$$\text{P(heart)} = \frac{13}{52} = \frac{1}{4}$$

Similarly, the probability of rolling an odd number on 1 die is P(odd) = $\frac{3}{6} = \frac{1}{2}$.

**Your problem is to find the probability of picking an ace or a diamond from a shuffled deck of cards.**

**Answer and explain.**

This problem involves probability.

The **probability** of an **event** happening, P(event), is equal to the number of **successful outcomes** divided by the number of **possible outcomes**.

$$P(\text{event}) = \frac{\#\, of\ successful\, outcomes}{\#\, of\ possible\, outcomes}$$

1. A student at Standard High explained that if the probability of flipping a coin once and getting a head is $\frac{\mathbf{1}}{\mathbf{2}}$, then the probability of getting a head if you flipped twice must be 1, since $\frac{\mathbf{1}}{\mathbf{2}} + \frac{\mathbf{1}}{\mathbf{2}} = 1$.

   Your problem is to find the *real* probability of flipping a coin twice and getting a head on one or both flips.

   Answer and explain.

2. Find the probability of flipping a coin and getting a tail followed by picking a diamond from a shuffled deck of cards, which can be represented as P(tail, diamond).

   Answer and explain.

Suppose you roll three standard dice and add up the results. The lowest sum you can get is 3 (by rolling three 1's), and the highest is 18 (by rolling three 6's).

1. Without doing any analysis, what sums would you expect to be the most likely? Why?

2. Find the probability of getting each of the three-dice sums. Describe any patterns that you find and explain them if you can.

For each problem, draw a RUG diagram representing.

1. Flipping a coin and tossing a die.

   a. Find P (H, 6)
   b. Find P (T, even)

2. Spinning a red, blue, white spinner with all three parts equal and flipping a coin.

   a. Find P (white, T)
   b. Find P (not white, H)

3. Tossing a die and spinning a red, blue, white spinner with all three parts equal.

   a. Find P (odd, red)
   b. Find P (3, not red)
   c. Find P (prime, white)

4. Flipping a coin and drawing a card from a standard deck.

   a. Find P (head, heart)
   b. Find P (tail, black)
   c. Find P (head, ace)

In the problems below, draw a RUG diagram and a TREE diagram to represent the situation. Then find the requested probabilities.

1. The result of flipping two coins.

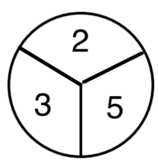
   a. Find P (2 heads)
   b. Find P (1 head, 1 tail)

2. The result of flipping three coins.

   a. Find P (exactly 2 heads)
   b. Find P (exactly one tail)
   c. Find P (at least 2 heads)
   b. Find P (at least one tail)

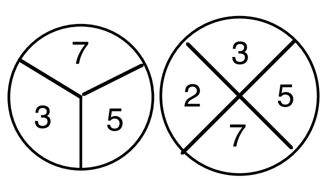
3. Spin the spinner (shown below) twice. Assume that each outcome is equally likely.

   a. Find P (6)
   b. Find P (at least 6)
   c. Find P (at most 6)
   d. Find P (at most 7)




4. Spin each spinner (shown below) once. Assume that each outcome is equally likely.

   a. Find P (odd)
   b. Find P (at least 8)
   c. Find P (8)
   d. Find P (less than 5)



You are a newspaper carrier who delivers the newspaper to people's homes every day of the week for $4 per week. One day Mrs. Reader offers you a different way of getting paid.

> "Suppose I put five bills in a bag and have you put your hand in the bag and pick one bill at random. The bills will be four $1 bills and a $20 bill. You keep what you get as payment for that week and you will be paid this way every week for the next fifty weeks. Of course, I will replace whatever you took the previous week so you always will be picking from four $1 bills and a $20 bill."

Your must decide whether to accept Mrs. Reader's offer or to continue collecting $4 each week.

Your problem, then is to decide whether you wiil get more money over the fifty week period by **drawing from the bag** or **being paid at $4 per week**

# and

you must tell how much more money you would **expect** to get over the fifty week period using the method you chose.

Justify (explain) your answer.

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> "Suppose I put five bills in a bag and have you put your hand in the bag and pick two bills at random. The bills will be three $1 bills, a $5 bill, and a $10 bill. You keep what you get as payment for that week and you will be paid this way every week for the next fifty weeks. Of course, I will replace whatever you took the previous week so you always will be picking from four $1 bills and a $20 bill."

Your must decide whether to accept Mrs. Reader's offer or to continue collecting $4 each week.

Your problem, then is to decide whether you wiil get more money over the fifty week period by **drawing from the bag** or **being paid at $4 per week**

# and

you must tell how much more money you would **expect** to get over the fifty week period using the method you chose.

Justify (explain) your answer.

Al and Betty were getting used to the idea of expected value and they were making some conjectures.

Al wanted to find the expected value if you rolled a die. He imagined rolling 600 times and figured that he would get about 100 ones, 100 twos, and so on.

So he did this computation:

$$100 \cdot 1 + 100 \cdot 2 + 100 \cdot 3 + 100 \cdot 4 + 100 \cdot 5 + 100 \cdot 6$$

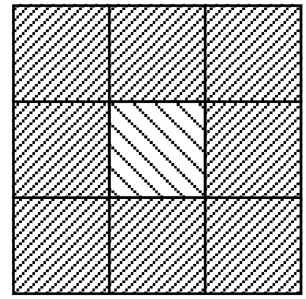
This gave a total of 2100 points for the 600 rolls. He then divided by 600 to get the average per roll, which came out to 3.5.

1. Betty tried it with 6000 rolls and got the same average. Explain why their averages are the same.

2. When Al saw the averages were the same, he decided he could find the average with only six rolls. Will he still get the same result? Explain.

3. Could you find the average with only one roll? Explain.

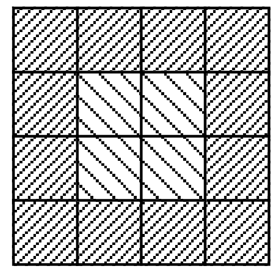
If a coin is flipped from across the room onto a rug and is equally likely to land on any part of the rug, find the following:



1. In the 3x3 rug, find

   P(inside square) =

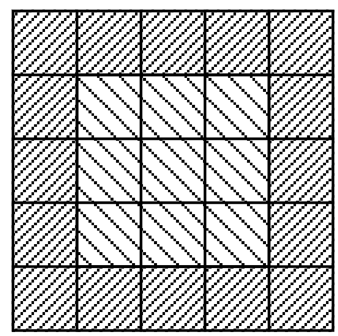
   P(border square) =



2. In the 4x4 rug, find

   P(inside square) =

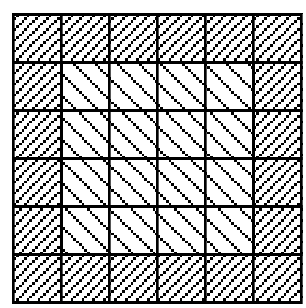
   P(border square) =



3. In the 5x5 rug, find

   P(inside square) =

   P(border square) =



4. In the 6x6 rug, find

   P(inside square) =

   P(border square) =

5. In an $n$ x $n$ rug, find

   P(inside square) =

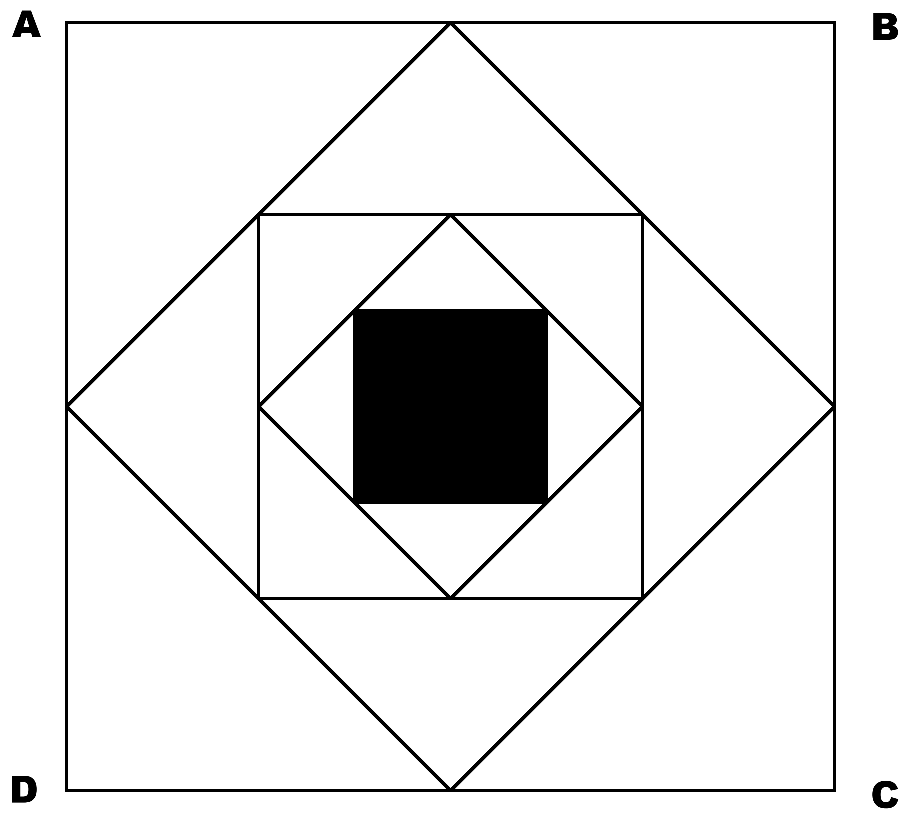
   P(border square) =

At a school fund-raiser, students set up a booth with this game.

> You start flipping a regular coin. Each time you get heads, you get a payoff of \$1. If you get tails, the game ends and you keep the money you have won so far. (If you get tails on your first flip, you get nothing.)
>
> Also, if you get 10 straight heads, you get your \$10 and the game ends and you are given a bonus of \$50. For example, if you flip four heads and then tails, you win \$4. If you flip 10 heads, you win \$60 altogether.

If the school charges \$2 to play and each of the 1024 students at the school plays five times, how much profit should the school expect to make altogether? Explain your reasoning.

Each smaller square is made by joining the midpoints of the sides of the larger surrounding square.
What is the probability of a dart that falls randomly on this diagram landing on black? Explain.



1. A parent wants to encourage his child to do well in school. He offers a "good report card incentive" based on major subject grades only. For each "A," his child will get $15. For each "B," his child will get $5. For each "C," "D," or "F", his child will get nothing. Assume that there are 5 major subjects and that each of the grades is equally likely. Draw a rug diagram and find the expected value of the payoff for the report card.

2. A restaurant is running a "Banana Split Promotion." The usual price for a banana split is $4. But for this special, slips of paper with various prices are folded and put into a balloon, which is then blown up and tied. A banana split customer picks a balloon, which is popped, and pays the price on the paper.
There are 8 balloons with prices as follows: 4 balloons contain a $4 slip, one balloon contains $3, one balloon contains $2, one balloon contains $1, and one balloon will reward you with a free banana split.
If you have a banana split every day, find the cost in the long run. (Assume that when a balloon is popped it is replaced with another balloon containg a slip with the same value.)

3. A casino introduces a new card game. It is played with a nine card deck consisting of the following cards:

2 of diamonds, 3 of clubs, 4 of spades, 5 of hearts, 6 of diamonds, 7 of clubs, 8 of spades, 9 of hearts, and 10 of spades.

You pay the dealer $5 to draw one card. You are paid by the dealer according to the following rules:

You win $1 for a spade, $2 for a diamond, $3 for a heart, and $4 for a club.
You win $1 for picking an even number and $2 for picking an odd number.
You win $5 if you pick a perfect square.

Thus, if you draw the 3 of clubs, you win $2 for picking odd and $4 for picking a club. Your payoff is $6.

Find your expected value for this game. In the long run do you win, do you lose, or is this a fair game (nobody wins or loses)?

Al and Betty are at the park flipping coins. Al gets a point if the coin is heads and Betty gets a point if the coin is tails. The first one to get 10 points wins a prize of $16.

But with Al leading by a score of 9 to 7, Al's parents and Betty's parents interrupt the game and Al and Betty are told they each have to go home. They decide that rather than finish the game at another time, they should just give out the prize now.

Al says that since he was leading, he should get the prize. Betty figures that each point should be worth $1, so Al should get $9 and she should get $7.

One of the parents suggests they should figure out the probability that each had of winning and divide the money according to that.

How should they divide the money if they take the parent's advice? Explain your results carefully.

# Cat and Mouse

You will need a die and a counter, as well as the game board.

Put the counter in the room where the mouse is. Throw the die. The mouse moves to the next room by following the rules:

Move in the direction O ⇒ if you throw an odd number.

Move in the direction E ⇒ if you throw an even number.

You win if the mouse gets the cheese. You lose if the mouse runs into the cat. Play the game 10 times and keep track of your wins and losses.

Based on your results, is this game fair?

Is the game different if you you play by tossing a coin and move in the O ⇒ direction if you toss a HEAD and move in the E ⇒ direction if you toss a tail? Play 10 games and count your wins and losses.

Based on your results, is this game fair?