The **mean** for a set of values is obtained by adding the numbers and dividing the result by the number of values that were added.

The **median** for a set of numbers is found by arranging the numbers in increasing or decreasing order, then choosing the middle number. For an even numberof values, the median is obtained by taking the two middle numbers and averaging them.

The **mode** for a set of values is the number that occurs most often. If all of the numbers are different, there is no mode. If two different numbers occur most often then there are two modes.

The **range** for a set of data is the difference between the highest and lowest values.

1. For the following set of numbers, find the mean, median and mode (if any),

   5, 8, 8, 11, 13

   Mean: _____ Median: _____ Mode: _____ Range: _____

2. For the following set of numbers, find the mean, median and mode (if any),

   15, 8, 12, 11, 4, 18, 14, 9

   Mean: _____ Median: _____ Mode: _____ Range: _____

3, Marvin bought 5 items at a mean (average) cost of 40¢.

   Find two different sets of prices that would give this result.

   ____________________________ ____________________________

4. Six students each worked on a term paper for their history class. They each spent a different whole number of hours on their papers. The average (mean) time spent by the students was 20 hours. The student who spent the least time spent 9 hours. The student who spent the most time spent 25 hours. Find three possibilities for the amounts of time the other 4 students spent on their papers.

   ____________________ ____________________ ____________________

Create a set of data that matches each set of restrictions.

1. Martin played in 5 basketball games. The mean number of points that he scored was 12. The median number of points he scored was 14.

____

2. Denise played in 5 basketball games also. Her mean number of points scored is 18. Her range of points scored was 9.

____

3. Six students kept track of how much time they spent watching TV in a week. Their mean time was 8 hours. Their mode time was 9 hours.

____

4. Ramon bought 5 books. Their mean cost was $15.20. Their median cost was $18. The most expensive book cost $24.

____

5. In a certain I.Q. test, an I.Q. of 100 is considered "average." Six students take this I.Q. test and have a mean score of 100,

   a. What is the greatest number of the six students who could have a score that is above "average?" Give a set of scores that demonstrates this.

   ____

   b. What is the smallest number of the six students who could have a score that is above "average?" Give a set of scores that demonstrates this.

   ____

6. Garrison Keillor claims that in Lake Wobegon, Minnesota, "All the children are above average." Is this possible? Explain.

Seven students were complaining about how much time they had to spend on schoolwork over their spring vacation. Their mean time was 15 hours. Make up a data set for each of the requirements given below. For each requirement, demonstrate that your data set meets the requirement by finding the mean, median, and mode.

1. The mean is larger than the median.

   ____

   Mean: _____ Median: _____ Mode: _____

2. The median is larger than the mean.

   ____

   Mean: _____ Median: _____ Mode: _____

3. The mean is larger than the mode.

   ____

   Mean: _____ Median: _____ Mode: _____

4. The mode is larger than the mean.

   ____

   Mean: _____ Median: _____ Mode: _____

5. The mode is larger than the median.

   ____

   Mean: _____ Median: _____ Mode: _____

6. The median is larger than the mode.

   ____

   Mean: _____ Median: _____ Mode: _____

7. The mean, median, and mode are equal.

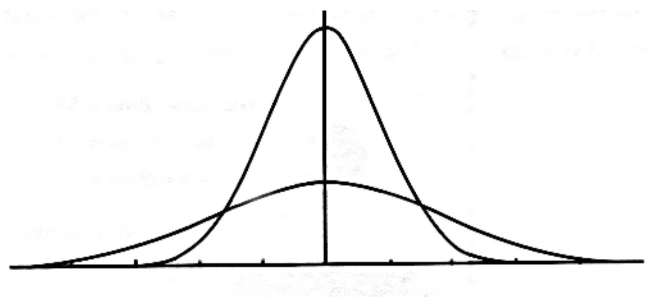
   ____

   Mean: _____ Median: _____ Mode: _____

What Is Standard Deviation?

The **standard deviation** of a set of data measures how "spread out" the data set is. In other words, it tells you whether all the data items bunch around close to the mean or if they are "all over the place."

The superimposed graphs below show two normal distributions with the same mean, but the taller graph is less "spread out." Therefore, the data represented by the taller graph has a smaller standard deviation.



Calculation of Standard Deviation

Here is a list of the steps for calculating standard deviation.

1. Find the mean.

2. Find the difference between each data item and the rnean.

3. Square each of the differences.

4. Find the average (mean) of these squared differences.

5. Take the square root of this average.

Organizing the computation of standard deviation into a table like the one on the next page can be very helpful. This table is based on a data set of five items: 5, 8, 10, 14, and 18. The mean for this data set is 11. The mean of a set of data is often represented by the symbol $\bar{x}$, which is read as "x bar."

The computation of the mean is shown below the table to the left. On the right below the table, step 4 of the computation of the standard deviation is broken down into two substeps: (a) adding the squares of the differences and (b) dividing by the number of data items.
The symbol usually used for standard deviation is the lower case form of the Greek letter sigma, written σ.

x	$\overline{x}$	$(x-\overline{x})^2$
5	-6	36
8	-3	9
10	-1	1
14	3	9
18	7	49

sum of the data items = 55
number of data items = 5
$\overline{x}$ (mean of the data items) = 11

sum of the squared differences = 104
mean of the squared differences = 20.8
σ (standard deviation) $= \sqrt{20.8} \approx 4.6$

Suppose you represent the mean as $\overline{x}$, use n for the number of data items, and represent the data items as x_1, x_2, and so on. Then the standard deviation can be defined by the equation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\overline{x})^2}{n}}$$

Standard Deviation and the Normal Distribution

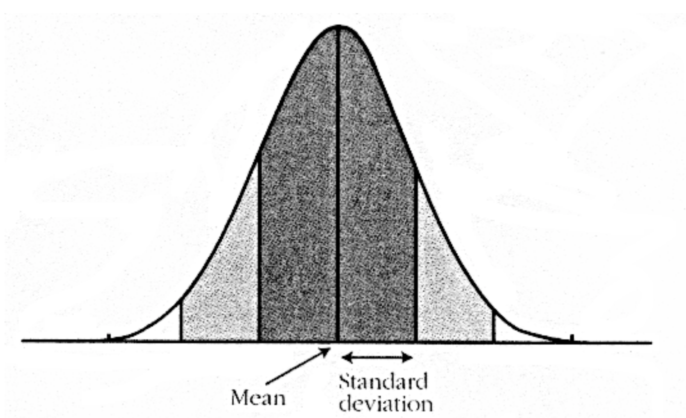
The normal distribution was identified and studied initially by a French mathematician, Abraham de Moivre (1667-1754). De Moivre used the concept of normal distribution to make calculations for wealthy gamblers. That was how he supported himself while he worked as a mathematician.

One of the reasons why standard deviation is so important for normal distributions is that there are some principles about standard deviation that hold true for any normal distribution. Specifically, whenever a set of data is normally distributed, these statements hold true.

Approximately 68% of all results are within one Standard deviation of the mean.

Approxinialcly 95% of all results are within two standard deviations of the mean.

These facts can he explained in terms of area, using the diagram "The Normal Distribution."



The Normal Distribution

In this diagram, the darkly shaded area stretches from one standard deviation below the mean to one standard deviation above the mean; it is approximately 68% of the total area under the curve.

The light and dark shaded areas together stretch from two standard deviations below the mean to two standard deviations ahove the mean, and constitute approxilliatcly 95% of' the total area under the curve.

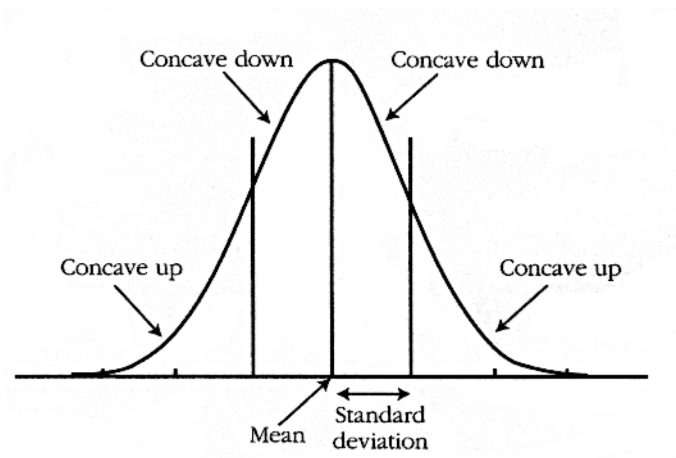
So standard deviation provides a good rule of thumb for deciding whether something is "rare."

Geometric Interpretation of Standard Deviation

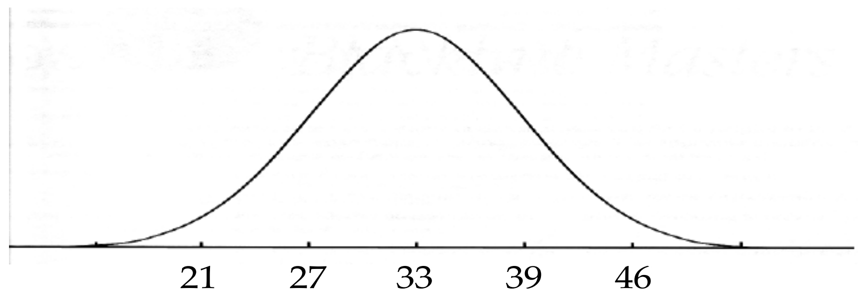
Geometrically, the standard deviation for a normal distribution turns out to be the horizontal distance from the mean to the place on the curve where the curve changes from being concave down to concave up.

In the diagram "Visualizing the Standard Deviation;' the center section of the curve, near the mean, is concave down, and the two "tails" (that is, the portions farther from the mean) are concave up.

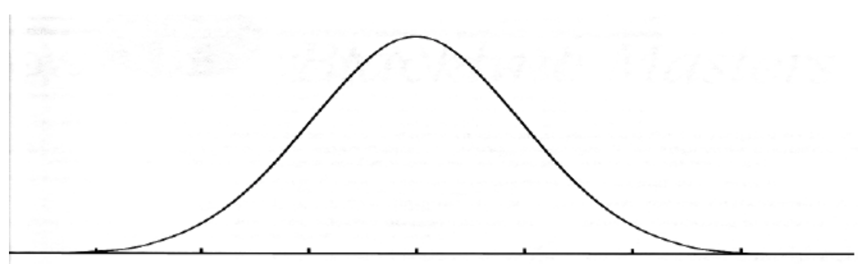
The two places where the curve changes its concavity, marked by the vertical lines, are exactly one standard deviation from the mean, measured horizontally.



The Normal Curve



1. For the data illustrated in the normal curve shown above, the mean is ________
2. The percent of scores between 27 and 39 is ________
3. The percent of scores between 21 and 27 is ________
4. The percent of scores above 46 is ________
5. The percent of scores falling outside the "normal" range is ________



In the curve shown above, the mean is 91.3 and the standard deviation is 8.4

6. Put the appropriate coordinates on the 5 labeled points on the axis above.
7. Is a score of 75 within the "normal" range for this sample?
8. Scores within one standard deviation from the mean are from ______ to ______

Make a frequency bar graph for each set of data. Then find the range and mean for the data.

1. Quiz scores 80, 75, 95, 95, 85, 90, 75, 75, 90, 95, 80, 95, 80, 90, 75

2. Height of students

Height (in inches)	Number of students
60	2
61	4
62	5
63	7
64	8
65	7
66	6
67	4
68	3
69	1
70	0

3. Number of seconds students can hold their breath. Each bar should represent a span of 5 seconds.

36, 45, 92, 73, 56, 34, 38, 32, 62, 54, 57, 73, 55, 46, 49, 48, 38, 51, 58, 84, 69, 39, 41, 48, 42

4. Number of pages in textbooks.

Subject	Number of pages
IMP 1	517
English Comp	433
English Lit	675
Social Studies	567
Biology	491
Spanish 1	457
French 1	423

For each set of data in the previous worksheet you have already found the range and the mean. Now find the standard deviation. The steps in finding the standard deviation are listed below.

To find the standard deviation:

1. Find the mean

2. Find the difference between each data item and the mean.

3. Square each of the differences.

4. Find the average (mean) of these squared differences.

5. Take the square root of this average.

Now answer the following questions.

1. Quiz scores

    a. What is the standard deviation?

    b. Are any of the quiz scores more than two standard deviations from the mean? If so, which ones?

2. Heights of students

    a. What is the standard deviation?

    b. Are any of the heights more than two standard deviations from the mean? If so, which ones?

    c. If all of these heights were from boys of the same age, what is the range of heights that would be considered “normal” or “usual?”

3. Number of seconds students can hold their breath

    a. What is the “normal” or “usual” range for how long students can hold their breath?

    b. How many students from this sample can hold their breath longer than usual?

4. Number of pages in textbooks

    a. Which textbooks are unusually long or short?

Are You Ambidextrous?

In this activity, you will compare the reflexes of your two hands. Here's how.

> Have a partner hold a ruler vertically between your thumb and forefinger, so that the lower end of the ruler is level with your fingers. Spread your thumb and forefinger as wide apart as possible. Your partner should hold the ruler from its upper end.
>
> Your partner will say "Drop" at the moment he or she drops the ruler. As the ruler falls past your fingers, you try to pinch it as quickly as you can.

1. Do the experiment with one of your hands 20 times, each time recording the place where you grab it.

2. Find the mean and the standard deviation of your data.

3. Draw a normal curve, with a horizontal scale, that has the same mean and standard deviation as your data. Show where the standard deviation marks are located.

4. Now do the ruler experiment one time with your other hand.

5. How many standard deviations from the mean was the experiment result from your "other" hand? What percent of the time do you think you would get a result that far from the mean using your "first" hand? Explain.

6. Do you think you are ambidextrous? Why or why not? (If you don't know what the word ambidextrous means, ask someone or look it up in a dictionary.)

You know that, in the long run, a normal die will come up 1 one-sixth of the time, 2 one-sixth of the time, and so on.

Thus, for example, with 36 rolls, we'd expect about six 1's six 2's, and so on.

1. Find the mean and standard deviation for the data represented by this "ideal result" for 36 rolls (six 1's, six 2's, and so on), and explain the computations you used.

2. Suppose the die were rolled 360 times, with the "ideal result" of 60 1's, 60 2's, and so on.

   a. Write down a guess about what the mean and the standard deviation would be for this set of data, and explain your guess.

   b. Actually find the mean and the standard deviation for this situation, and explain the computations you used.

   c. Are the actual mean and the actual standard deviation the same as in Question 1? Explain why you think they came out the same or different.

3. Now suppose you used a pair of fair dice, rolling them together and each time finding the sum of the numbers on the two dice.

   a. What would be the "ideal result" for these sums if the pair of dice were rolled 36 times? (That is, how many 12's would you get, how many 11's, and so on, as the sum of the pair of dice?)

   b. Write down a guess about what the mean and the standard deviation would be for this "ideal result" of 36 sums for a pair of dice, and explain your guess.

   c. Find the actual mean for this "ideal result." How does it compare to your mean in Question 1?

   d. Find the actual standard deviation for this "ideal result." How does it compare to your standard deviation in Question 1?

1. Find the mean and standard deviation for the following sets of numbers:

a.	5, 8, 12, 14, 16	mean: ____	standard deviation: ____
b.	8, 11, 15, 17, 19	mean: ____	standard deviation: ____
c.	3, 7, 11, 14, 15	mean: ____	standard deviation: ____
d.	11, 15, 19, 22, 23	mean: ____	standard deviation: ____
e.	10, 16, 24, 28, 32	mean: ____	standard deviation: ____
f.	24, 33, 45, 51, 57	mean: ____	standard deviation: ____

2. What happens to the mean when the same number is added to the five original numbers? (Hint: look at the sets of numbers in problem 1)

3. What happens to the standard deviation when the same number is added to the five original numbers?

4. What happens to the mean when each of the five original numbers is multiplied by the same number?

5. What happens to the standard deviation when each of the five original numbers is multiplied by the same number?

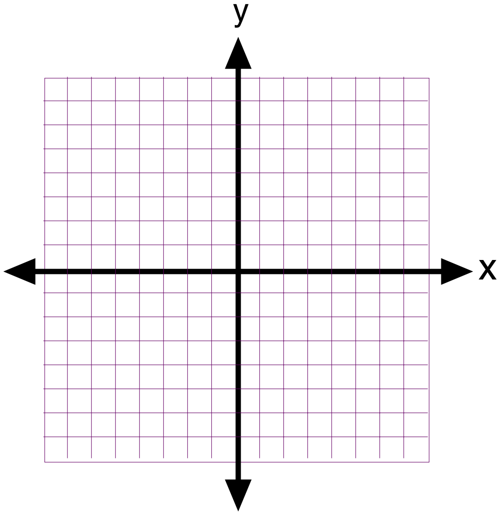
6. Make up a set of five numbers with mean of 8 and a standard deviation of 1.

7. Make up a set of five numbers with mean of 6 and a standard deviation of 1.

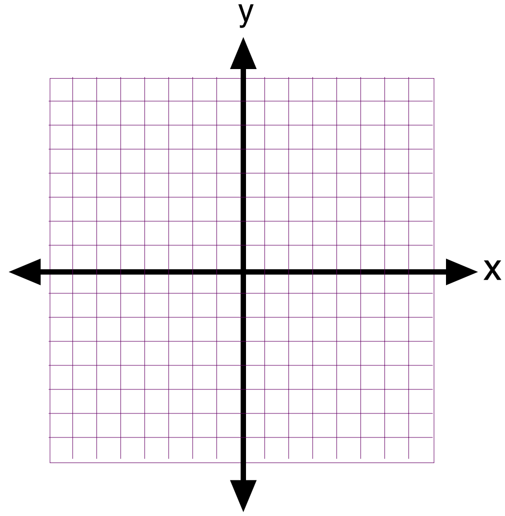
8. Make up a set of five numbers with mean of 10 and a standard deviation of 2

9. Make up a set of five numbers with mean of 11 and a standard deviation of 3.

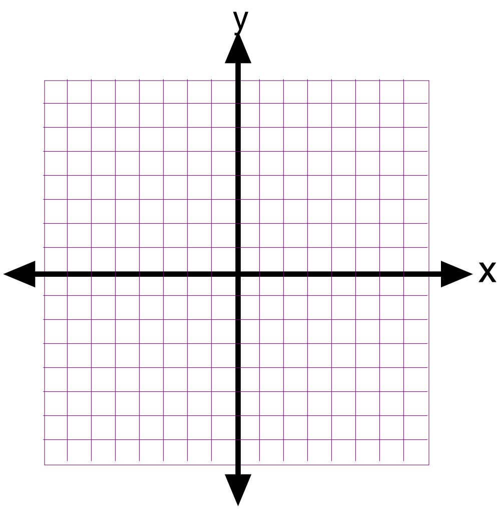
1. Graph $y = 6$ and $y = -2$



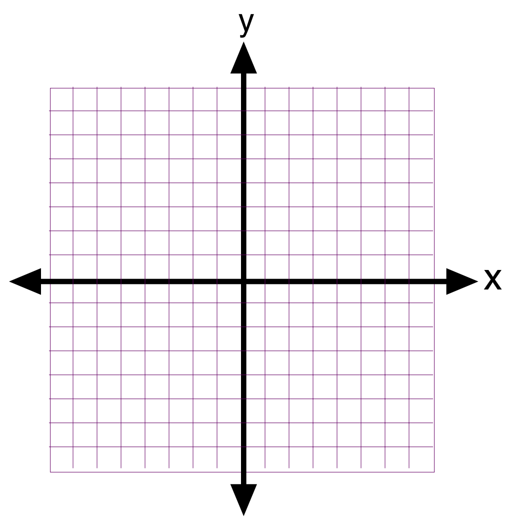
2. Graph $y = x$ and $y = x - 4$



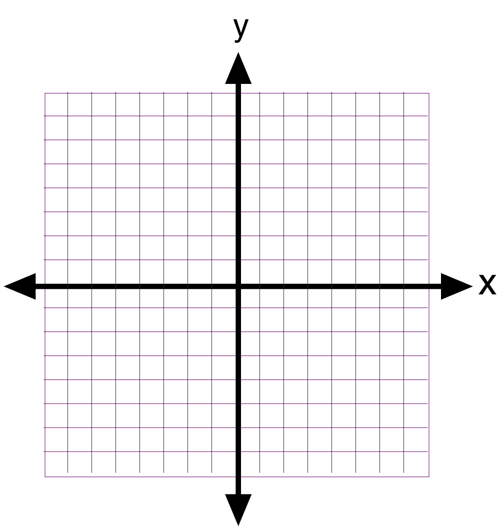
3. Graph $y = -\frac{3}{4}x + 4$ and $y = -\frac{3}{4}x - 1$



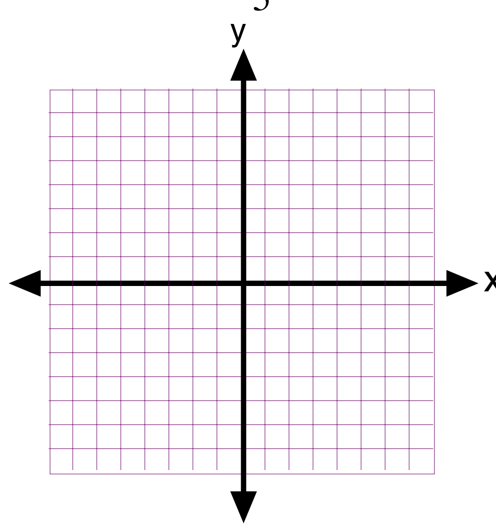
4. Graph $y = \frac{2}{3}x + 2$ and $y = \frac{2}{3}x - 2$



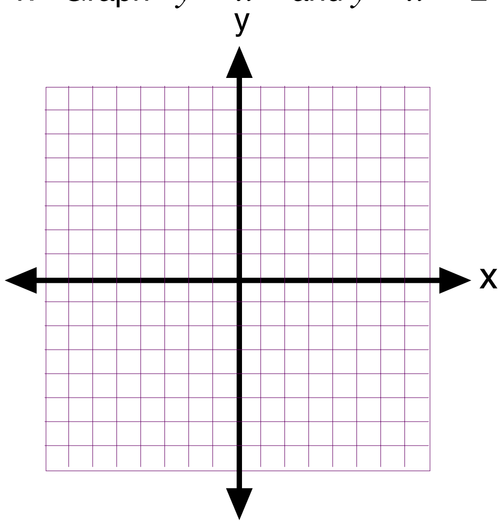
5. Graph $y = 2x - 5$ and $y = 2x$



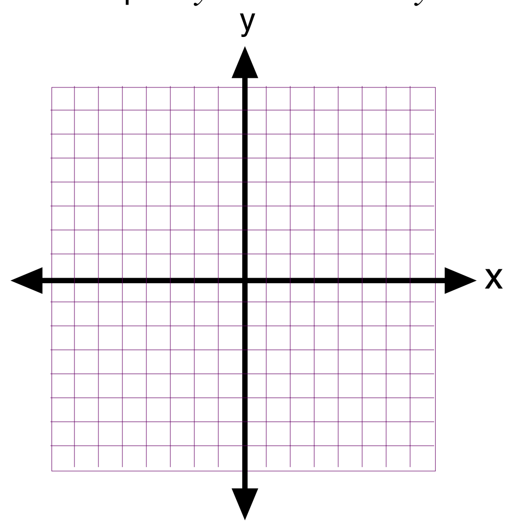
6. Graph $y = -\frac{4}{5}x + 1$ and $y = -\frac{4}{5}x +$



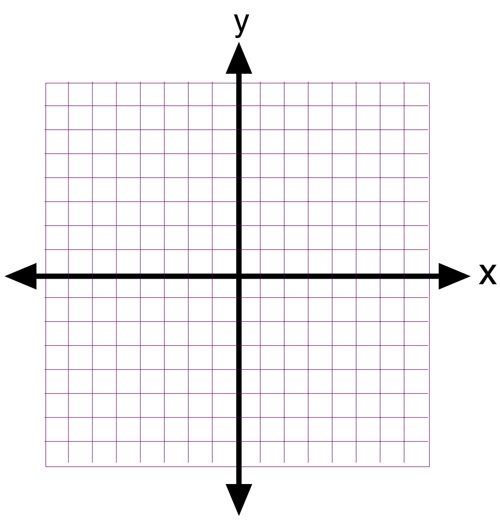
1. Graph $y = x^2$ and $y = x^2 - 2$



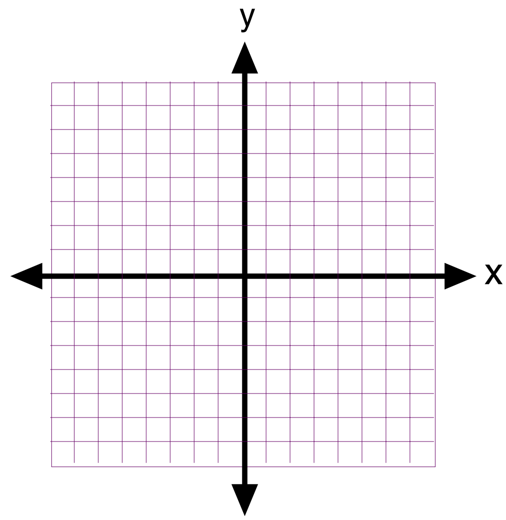
2. Graph $y = x^2 - 3$ and $y = x^2 - 6$



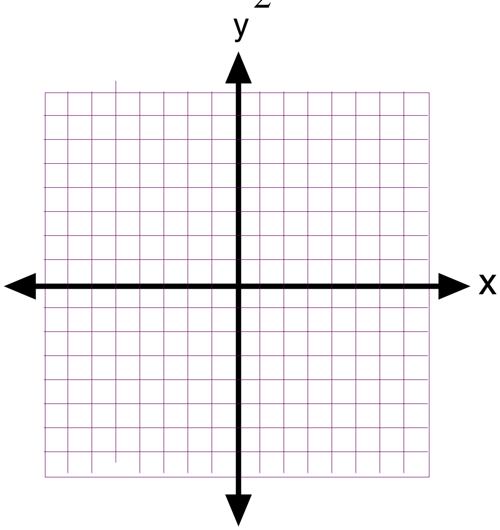
3. Graph $y = \frac{1}{2}x^2$ and $y = 2x^2$



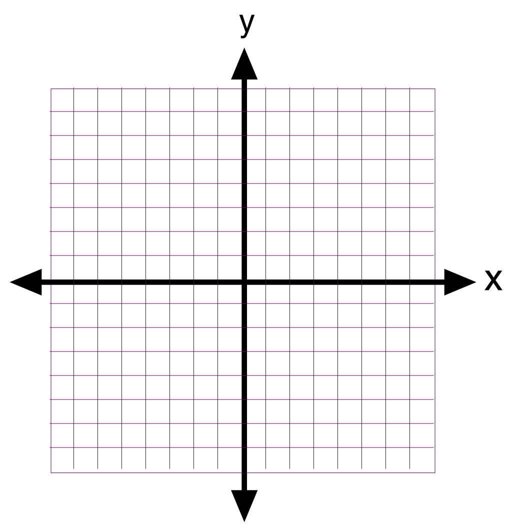
4. Graph $y = -x^2$ and $y = -x^2 + 4$



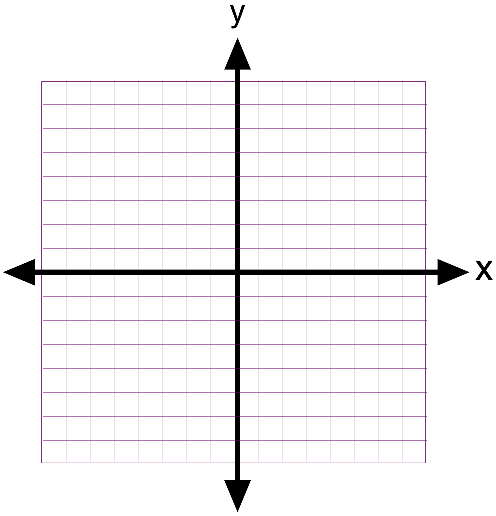
5. Graph $y = -\frac{1}{2}x^2$ and $y = -2x^2$



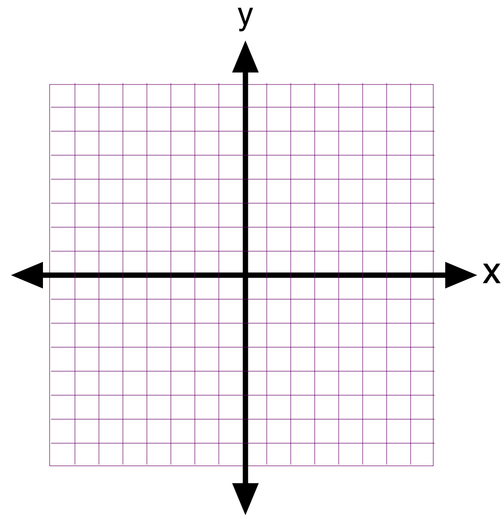
6. Graph $y = x^2 - 4$ and $y = -x^2 + 4$



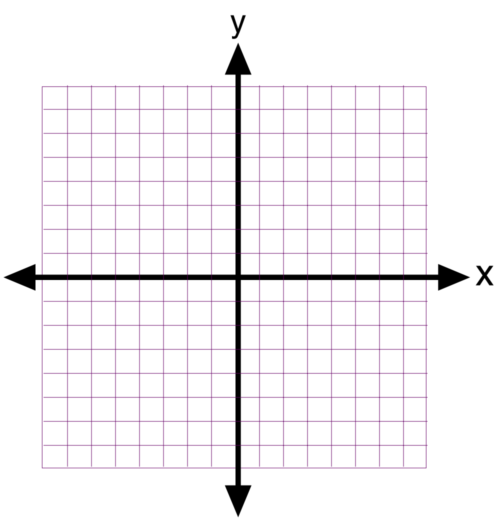
1. Graph $y = \sqrt{x}$ and $y = \sqrt{x} + 3$



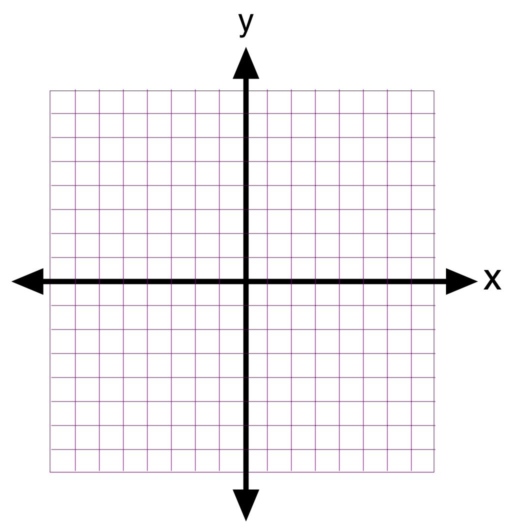
2. Graph $y = \sqrt{x} - 3$ and $y = \sqrt{x} + 2$



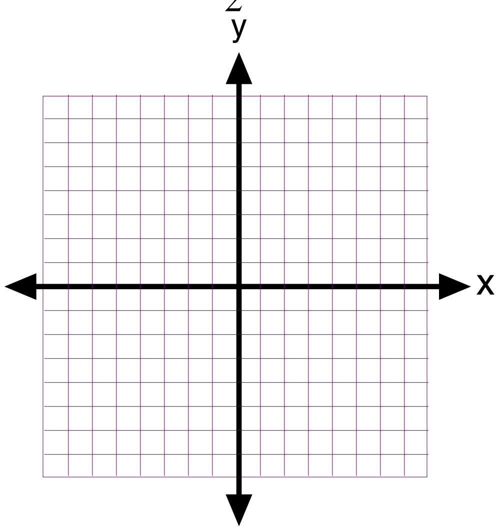
3. Graph $y = \frac{1}{2}\sqrt{x}$ and $y = 2\sqrt{x}$



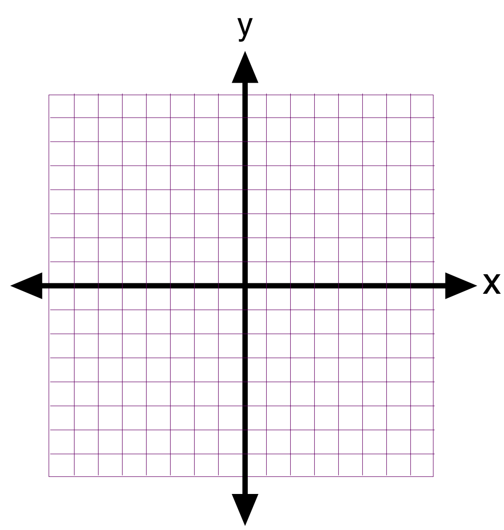
4. Graph $y = -\sqrt{x}$ and $y = -\sqrt{x} + 5$



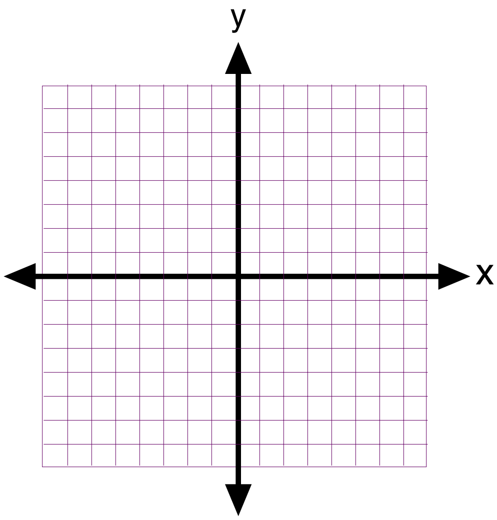
5. Graph $y = -\frac{1}{2}\sqrt{x}$ and $y = -2\sqrt{x}$



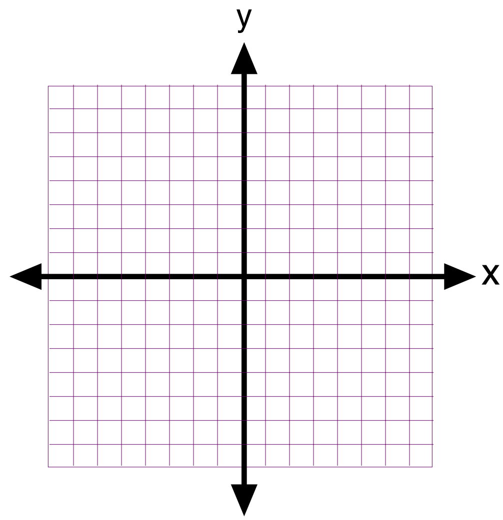
6. Graph $y = \sqrt{x} - 4$ and $y = -\sqrt{x} + 4$



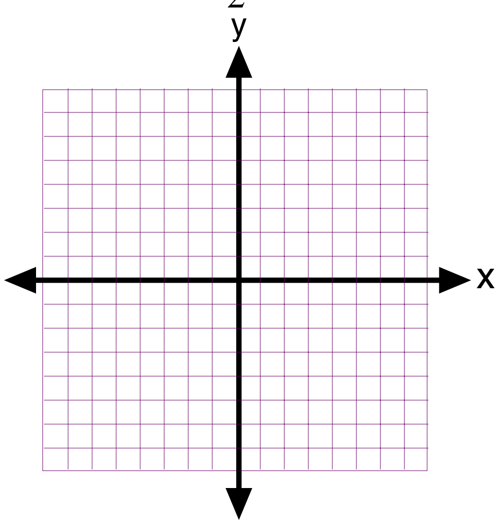
1. Graph $y = -3\sqrt{x}$



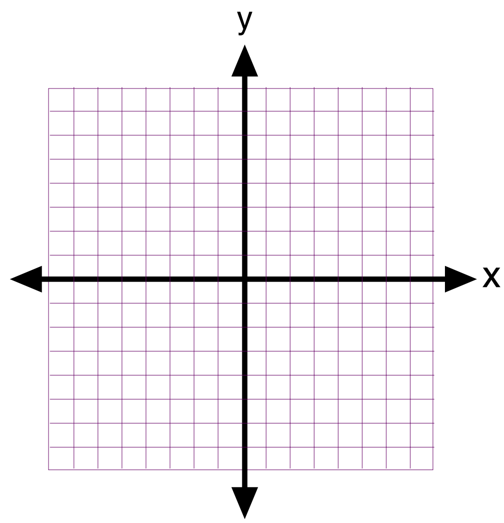
2. Graph $y = x^2 - 4$



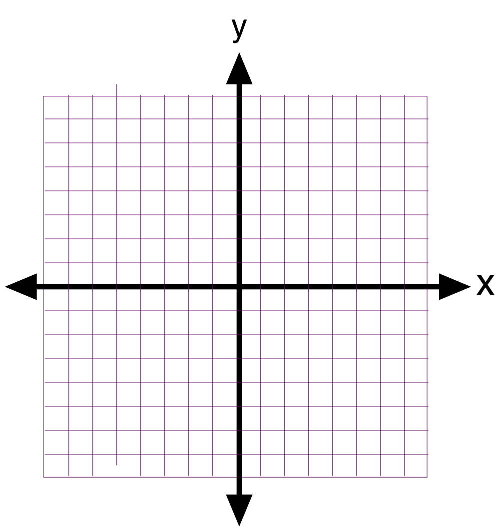
3. Graph $y = -\frac{1}{2}x + 3$



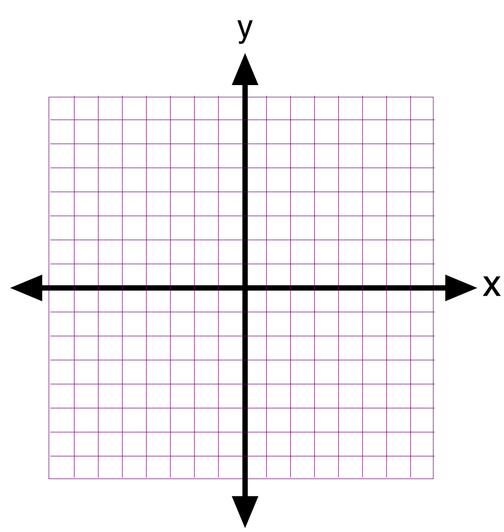
4. Graph $y = \sqrt{x} - 3$



5. Graph $y = -2x^2 + 5$

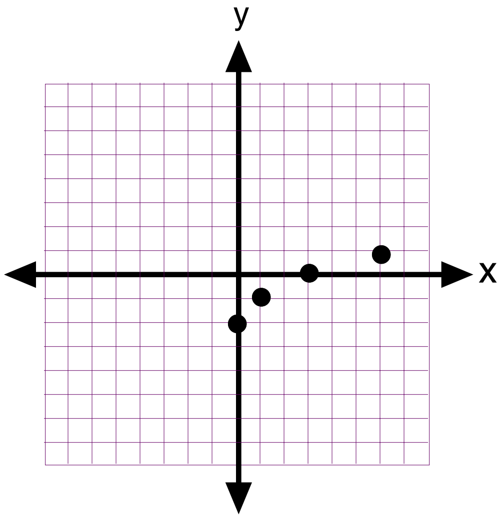


6. Graph $y = 2x - 3$

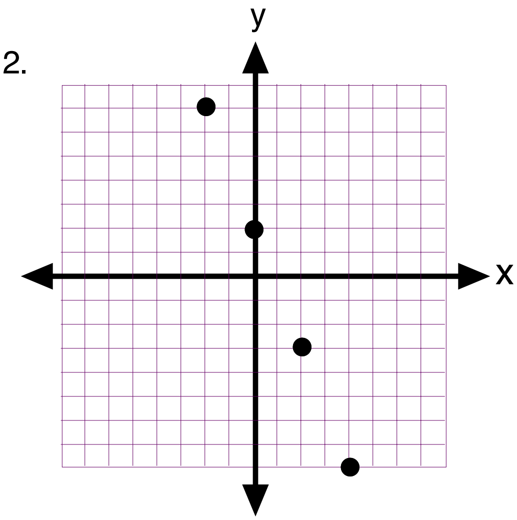


Find the equation that best fits each set of points.

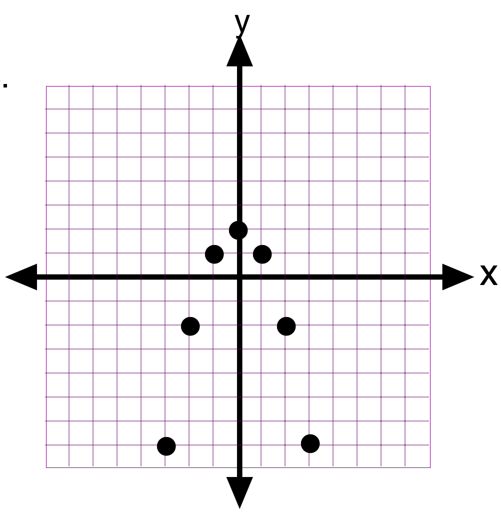
1.



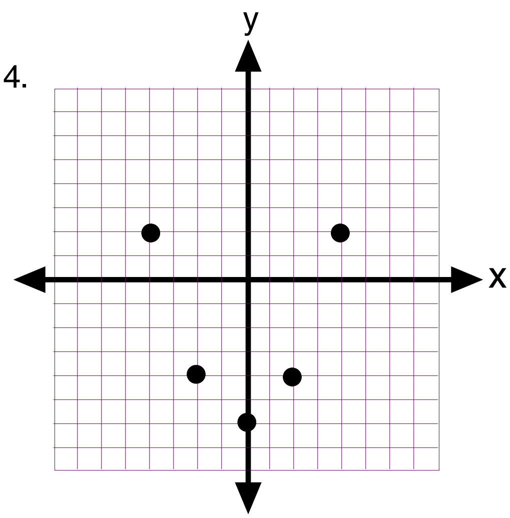
2.



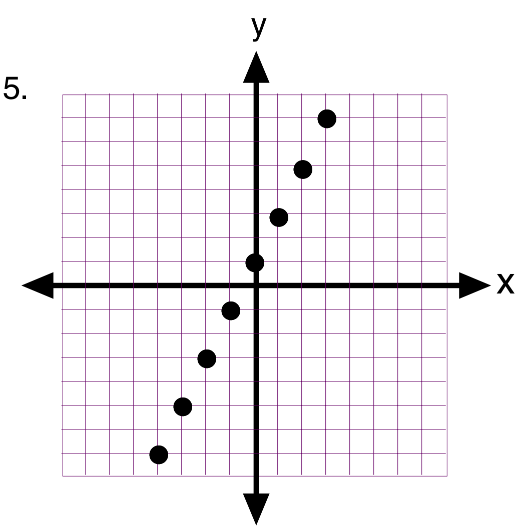
3.



4.



5.



6.