1. A pole that is 10 feet high casts a 6 foot shadow.

   a. How long a shadow will a 40 foot high pole cast?

   b. If a pole has a 15 foot shadow, how high is the pole?

2. A 10 foot tall tree casts a 6 foot shadow.

   a. Another tree casts a 15 foot shadow. How tall is it?

   c. How long a shadow will a 32 foot tall tree cast?

3. A map is scaled so that 1 inch on the map is equal to 2 miles.

   a. How many inches on the map is a 12 mile distance?

   b. What distance is represented by 17 inches on the map?

4. A map is scaled so that 2 inches on the map is equal to 3 miles.

   a. How many inches on the map is a 12 mile distance?

   b. What distance is represented by 30 inches on the map?

   c. What distance is represented by 17 inches on the map?

5. A map is scaled so that 5 inches on the map is equal to 8 miles.

   a. How many inches on the map is a 12 mile distance?

   b. How many inches on the map is a 30 mile distance?

   c. What distance is represented by 8 inches on the map?

   d. What distance is represented by 17 inches on the map?

6. A 13 foot tall tree casts a 5 foot shadow.

   a. Another tree casts a 12 foot shadow. How tall is it?

   b. How long a shadow will a 32 foot tall tree cast?

Properties of Proportions - Worksheet

Find the value of the given variable:

1. $\frac{x}{8} = \frac{12}{18}$

2. $\frac{x}{12} = \frac{60}{45}$

3. $\frac{c}{6} = \frac{12}{15}$

4. $\frac{q}{56} = \frac{15}{14}$

5. $\frac{8}{d} = \frac{40}{30}$

6. $\frac{1}{2} = \frac{z}{25}$

7. $\frac{3x}{21} = \frac{5}{3}$

8. $\frac{x}{9} = \frac{16}{x}$

9. $\frac{9}{12} = \frac{x-1}{4}$

10. $\frac{x}{6} = \frac{1}{2}$

Find the value of x.

1. $\frac{9}{x} = \frac{3}{4}$

2. $\frac{x}{2} = \frac{3}{5}$

3. $\frac{3}{2} = \frac{x}{6}$

4. $\frac{3}{2} = \frac{x+1}{8}$

5. $\frac{x}{-2} = \frac{3}{4}$

6. $\frac{6}{-5} = \frac{x}{6}$

7. $\frac{x+3}{6} = \frac{2}{3}$

8. $\frac{4}{5} = \frac{7}{2x+1}$

9. $\frac{8}{3} = \frac{5x}{6}$

10. $\frac{5}{3} = \frac{x-1}{-2}$

11. $\frac{3x+1}{6} = \frac{2}{3}$

12. $\frac{3x}{2} = \frac{5}{6}$

13. $\frac{18}{4x-3} = \frac{1}{2}$

14. $\frac{x}{2} = \frac{4}{3}$

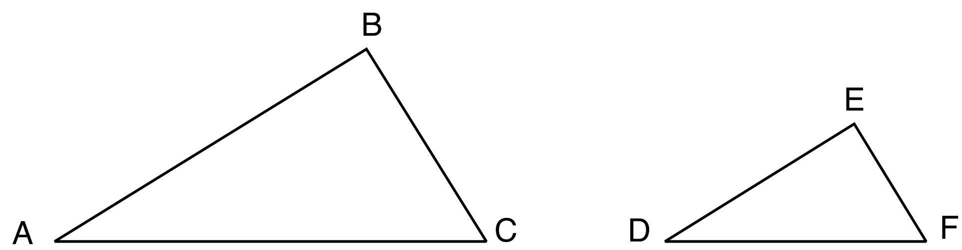
15. $\frac{3}{5} = \frac{2}{4x+1}$

16. $\frac{-3}{4} = \frac{x}{-8}$

Polygons and Similarity

A **polygon** is named by naming its **vertices** in order. For example, the polygon at the right can be called "triangle ABC" or "triangle CBA" or "triangle BCA" or "triangle ACB" or "triangle CAB" or ""triangle BAC."

In the two triangles shown below, *angle A* ***corresponds to*** *angle D, angle B* ***corresponds to*** *angle E, and angle C* ***corresponds to*** *angle F.*



If two polygons are **similar**, it means they have the same shape. They might or might not be the same size.

Two polygons are similar if:
1. Corresponding angles are equal, and
2. Corresponding sides are proportional.

To indicate that triangle ABC is similar to triangle DEF, we write $\Delta ABC \sim \Delta DEF$. The symbol "~" is read "is similar to." We could also write $\Delta CAB \sim \Delta FDE$, but we could **not** write $\Delta ABC \sim \Delta FDE$. When we write $\Delta CAB \sim \Delta FDE$, we are also saying that $\angle C$ corresponds to $\angle F$, $\angle A$ corresponds to $\angle D$, and $\angle B$ corresponds to $\angle E$. In other words, the first vertex of the first triangle must correspond with the first vertex of the second triangle, and so on.

Also, according to the two conditions stated above, $\angle A = \angle D$, $\angle B = \angle E$, and $\angle C = \angle E$,

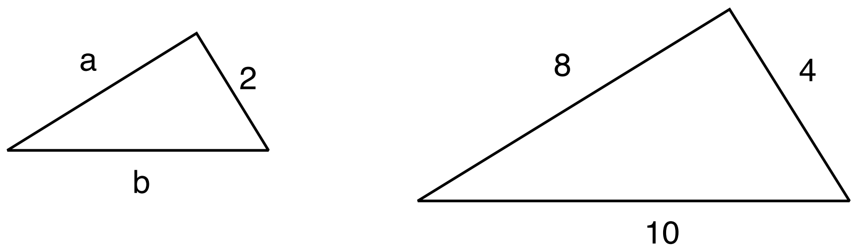
and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$.

In each of the pairs of figures shown below, assume that the figures are similar. Then

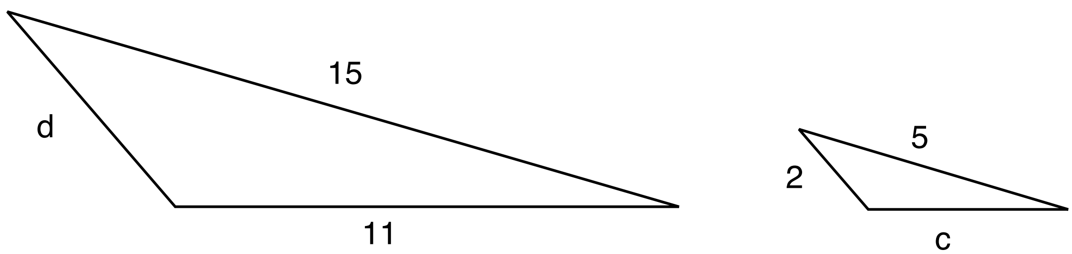
a. Write the proportion relating the sides of the figures.

b. Solve the proportions to find the missing lengths.

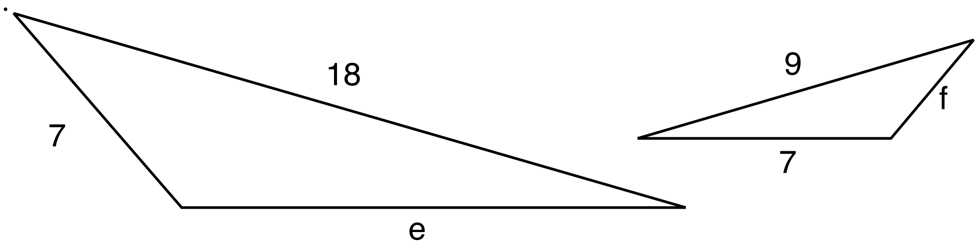
1.



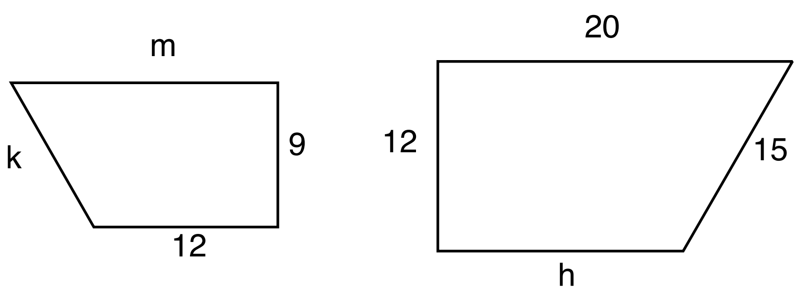
2.



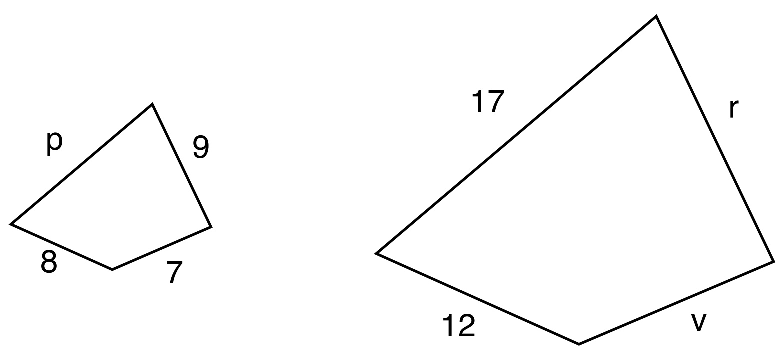
3.



4.



5.



<u>Answers</u>

1. a ______

   b ______

2. c ______

   d ______

3. e ______

   f ______

4. h ______

   k ______

   m ____

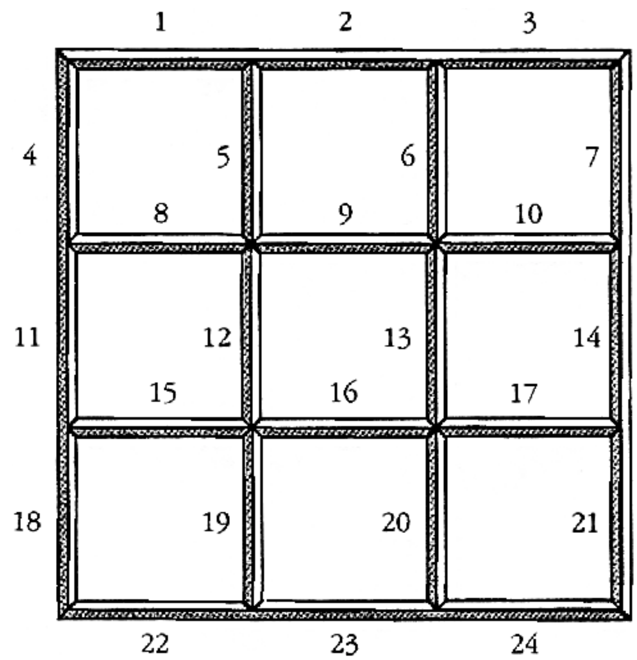
5. p ______

   r _______

   v ______

An N-by-N Window

This problem involves finding a formula.



The diagram to the right shows the frame for a window 3 feet by 3 feet.

The frame is made of wood strips that separate the glass panes. Each glass pane is a square that is 1 foot wide and 1 foot tall.

As the numbering in the diagram shows, it would take 24 feet of wood strip to build a frame for a window 3 feet by 3 feet.

Your task is to develop a formula for the total length of wood strip needed to build square windows of different sizes.

You may want to do this by gathering data and making an In-Out table from different examples. Or you may prefer to study the window looking for insights that lead directly to a formula. (Or you may do a combination of both.)

In either case, generalize the problem to a window N feet by N feet. As usual, include all drawings, In-Out tables, and graphs.

Sin, Cos, and Tan Buttons Revealed

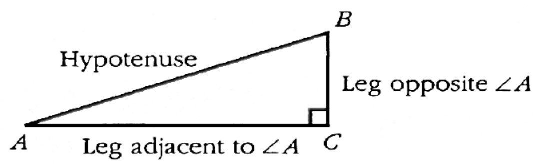
Did you ever wonder what those keys on your calculator that say "sin," "cos," and "tan" are all about? Well, here's where you find out.

You've seen that, whenever two right triangles have another angle in common, the triangles must be similar, and so the corresponding ratios of lengths of sides within those triangles are equal.

These ratios depend only on that common acute angle, and each ratio of lengths within the right triangle has a name. The study and use of these ratios is part of a branch of mathematics called **trigonometry.**

Suppose you are given an acute angle (in other words, an angle between 0° and 90°).

You can create a right triangle in which one of the acute angles is equal to that given angle. Suppose you label that triangle as shown in the diagram below, so that $\angle A$ is equal to the acute angle you started with.



The trigonometric ratios are then defined as explained on the following pages. The principles of similarity guarantee that these ratios will be the same for *every* right triangle that has an acute angle the same size as $\angle A$.

Sine of an Angle

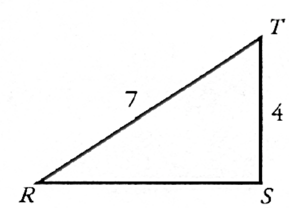
The **sine** of $\angle A$ is the ratio of the length of the leg opposite $\angle A$ to the length of the hypotenuse. The sine of $\angle A$ is abbreviated as **sin** A. For example, in ΔRST below, the leg opposite $\angle R$ has length 4, and the hypotenuse has length 7, so $\sin R = \frac{4}{7}$.

In summary

$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}}$$

or simply,

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$



Cosine of an Angle

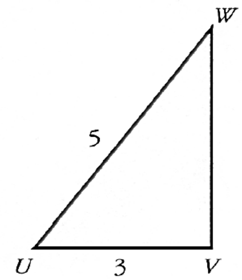
The **cosine** of $\angle A$ is the ratio of the length of the leg adjacent to $\angle A$ to the length of the hypotenuse. The cosine of $\angle A$ is abbreviated as **cos** A. For example, in ΔUVW below, the leg adjacent to $\angle U$ has length 3, and the hypotenuse has length 5, so $\cos U = \frac{3}{5}$.

In summary

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}}$$

or simply,

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$



Tangent of an Angle

The **tangent** of $\angle A$ is the ratio of the length of the leg opposite $\angle A$ to the length of the leg adjacent to $\angle A$. The tangent of $\angle A$ is abbreviated as **tan** $\boldsymbol{A}$. For example, in ΔHKL on

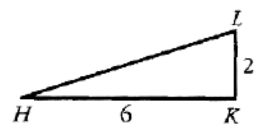
s

In summary

$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A}$$

or simply,

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$



Trigonometric Functions on a Calculator

Any scientific calculator or graphing calculator has keys that will give you the values of these functions for any angle.

In some calculators, you enter the size of the angle and then push the appropriate trigonometric key, while for other calculators, you do the opposite.

Caution: You have been measuring angles using ***degrees*** as the unit of measurement, but there are other units for measuring angles. Most calculators that work with trigonometric functions have a ***mode*** key that you can set to "deg."

The Other Ratios

There are three other ratios of side lengths within a right triangle, in addition to the sine, the cosine, and the tangent. These other ratios are used less often and usually do not have their own calculator keys.

Each is the reciprocal of one of the three ratios already defined. Here are the definitions of those other ratios.

$$\text{cotangent } A = \frac{1}{\text{tangent } A}$$

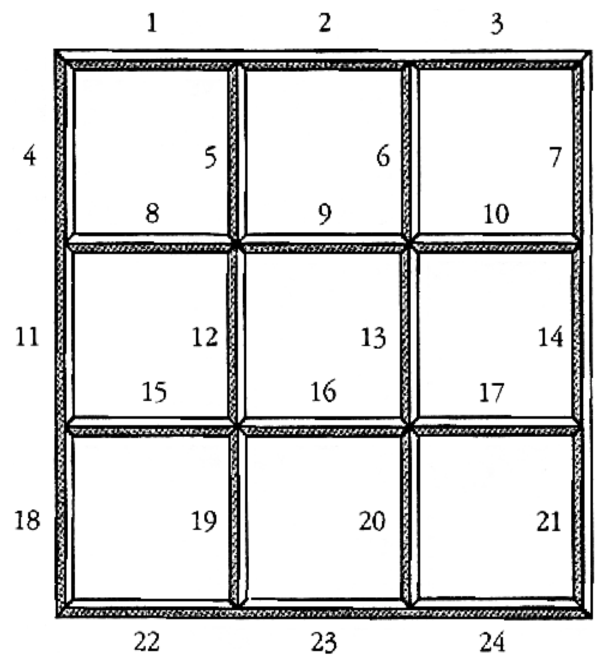
$$\text{secant } A = \frac{1}{\text{cosine } A}$$

$$\text{cosecant } A = \frac{1}{\text{sine } A}$$

They are abbreviated, respectively, as **cot** $\boldsymbol{A}$, **sec** $\boldsymbol{A}$, and **csc** $\boldsymbol{A}$.

More About Windows

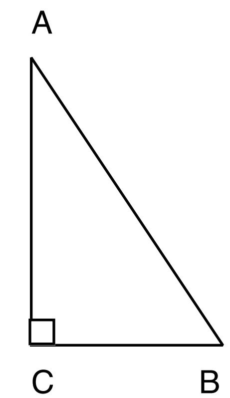
Try to generalize the formula you got for a square window in Shadows 6: *An N-by-N Window* to an arbitrary rectangular window frame. That is, get a formula in terms of M and N for the amount of wood strip needed for the fi-ame of an M-by-N window.



As in *Shadows 6: An N-by-N Window*, you may want to gather data about a variety of examples into an In-Out table and then look for a pattern in your data. If so, once you gather data, look for an algebraic rule that describes your table.

Or you may prefer a more analytic approach, in which you reason through why such a window frame should use a particular amount of wood strip. if you find a rule this way, verify it using some examples.

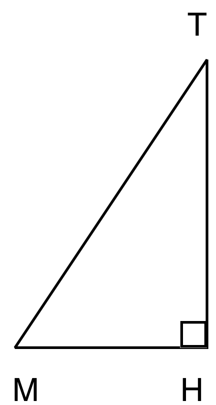
TRIANGLE TRIGONOMETRY



Find all lengths to nearest tenth.

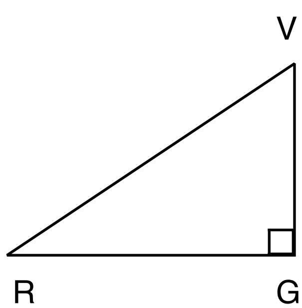
1. AB = 12, ∠B = 50°, AC = ____ BC = ____

2. BC = 8, ∠A = 43°, AC = ____ AB = ____



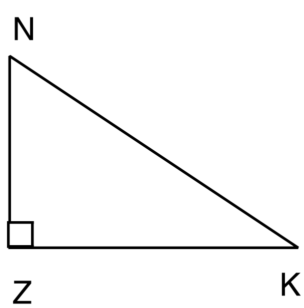
3. HT = 5, ∠M = 67°, MT = ____ HM = ____

4. MT = 13, ∠T = 21°, HT = ____ HM = ____



5. VR = 9, ∠V = 74°, GR = ____ GV = ____

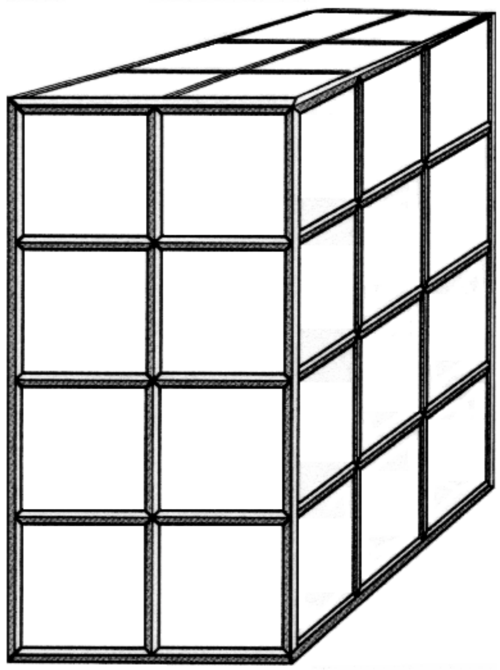
6. GR = 26, ∠R = 13°, RV = ____ GV = ____



7. NZ = 17, ∠N = 79°, KZ = ____ KN = ____

8. KZ = 4, ∠K =60°, NZ = ____ KN = ____

Crates



In Shadows 6: *An N-by-N Window* and in Shadows 9: *More About Windows*, you looked for formulas for the amount of wood strip needed to create a window frame, Now suppose that instead of building a window frame, you are building a frame for a wooden crate, such as the one shown at the left. Keep in mind that the wood strip is just used to make aframe for the crate. Cardboard has been placed between the strips to make the crate into a solid box. Because of the cardboard, you can't see the bottom or the back sides of the crate in the diagram.

First find the amount of wood strip needed if the crate is 2 feet wide, 3 feet long, and 4 feet high, as shown at the left.

Remember that only three sides of the crate are shown here, so you'll need to imagine the other three sides. Also remember that some of the wood strips are shared by two sides of the crate.

Now look for a general formula, using a crate that is *w* feet wide, *l* feet long, and *h* feet high.

For each problem, make a right triangular drawing labeled correctly with the given information and a variable for what you are trying to find. FORM A TRIG EQUATION, using sin, cos, or tan; THEN, write an equation that would find your variable. Using a calculator, find the solution to the question.

1. When the angle of elevation of the sun is 40 degrees, a building casts a shadow of 800 feet. How tall is the building?

2. How far up a vertical wall does a 12 foot ladder reach if the angle it makes with the ground is 75 degrees?

3. The operator of a lighthouse spots a sailboat on a line that makes an 8 degree angle with the horizontal. If the top of the lighthouse is 25 meters above sea level, the sailboat is how far away from the base of the lighthouse?

4. A support cable for a 50 ft. tower is to make a 60 degree angle with the ground. How long must the cable be? (The cable goes to the top of the tower).

Producers for the Tonight Show have positioned spot lights at opposite corners of the auditorium. The host will be standing center stage at a distance of 300 feet from each spotlight. The beams form a 60 degree angle with each other. How far apart are the spotlights?