

PROBLEMS RELATING TO TOPICS - ANSWERS

1. More than 53,000,000 miles. You can ESTIMATE the answer with a class by showing the multiplication as $.003 \text{ in} \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \dots \cdot 2$ or $.003 \cdot 2^{50}$, then, by using the laws of exponents, as $.003 \cdot 2^{10} \cdot 2^{10} \cdot 2^{10} \cdot 2^{10} \cdot 2^{10}$.

Since $2^{10} = 1024$, estimate that $2^{10} \approx 1000$. Then rewrite as

$$\frac{3}{1000} * 1000 * 1000 * 1000 * 1000 * 1000 \text{ in.}$$

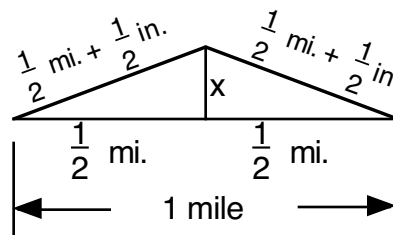
This is equal to $3 * 1000 * 1000 * 1000 * 1000 \text{ in.}$ Now convert inches to feet.

$$3 * 1000 * 1000 * 1000 * 1000 \text{ in.} \cdot \frac{1 \text{ ft.}}{12 \text{ in.}} = 250 * 1000 * 1000 * 1000 \text{ ft.}$$

Now approximate 1 mile as 5000 feet and convert to miles.

$$250 * 1000 * 1000 * 1000 \text{ ft.} * \frac{1 \text{ mi.}}{5000 \text{ ft.}} = 50,000,000 \text{ mi.}$$

2. Can't be done. Sixty miles per hour is equal to one mile per minute. Two miles takes two minutes. One mile at 30 miles per hour takes two minutes. No time is left for the second mile.
3. Circumference of the earth is about 8000π miles, or $2\pi \cdot 4000$ miles. The new circumference is $2\pi (4000 \text{ miles} + 1 \text{ yard})$ or $8000\pi \text{ miles} + 2\pi \text{ yards}$. The additional string is $2\pi \text{ yards}$, or approximately 6.3 yards of string.
4. Using the Pythagorean Theorem with all units converted to inches, x is approximately 178 inches, which is more than 14 feet.



5. Think of a triangle with sides of lengths 10, 20, and 30. Obviously these lengths cannot form a triangle. The only change that can be made in units of ten to form a triangle is to change the 10 to 40.
6. Fifty-nine years old. There was no year "0."
7. Feathers. Feathers are weighed by Avoirdupois weight (which we commonly use), in which a pound is 7,000 grains. Gold and other precious metals are weighed by Troy weight, in which a pound is 5,760 grains.
9. ABCD is a rectangle. The diagonals are congruent. Therefore $AB = OB = \text{radius} = 6$.
10. It is impossible to have a 5-5-12 triangle. Therefore, the missing side must be 12.

8. The key to this problem lies in the distances.

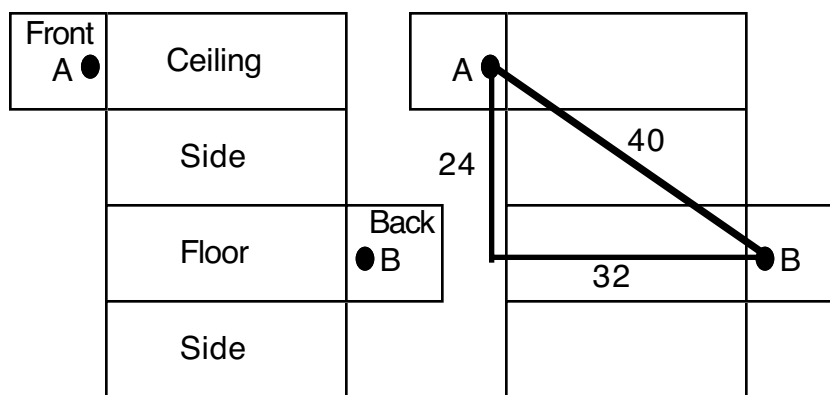
| DISTANCES | TRAIN | MAN |
|----------------|---------|---------|
| To point B | $x + y$ | $5/8 y$ |
| To point A | x | $3/8 y$ |
| By subtraction | y | $1/4 y$ |

The train crosses the entire bridge (y) in the same amount of time the man crosses $1/4$ of the bridge. Hence, the man's speed is $1/4$ of the train's speed. The man can run 15 mph.

11. You can get the smallest number of weights if your choice of weights only permits one way to measure a particular weight. For instance, if you have weights of 1, 2, and 3 ounces, you can weigh a 2 ounce item by using only the 2 ounce weight or by using the 1 and 3 ounce weights on opposite sides of the scale. If you eliminate the 2 ounce weight, you can still weigh 1, 2, 3, and even 4 ounce items with no duplication.

To avoid duplication, each new weight should be $2 \cdot (\text{sum of previous weights}) + 1$. If you start with a 1 unit weight, the next weight should be $2 \cdot (1) + 1$, or 3 units. The next weight should be $2 \cdot (1 + 3) + 1$, or 9 units. The next weight should be $2 \cdot (1 + 3 + 9) + 1$, or 27 units. You will also notice that these weights are powers of 3.

12. You may think that the ant should walk up the wall, across the ceiling, and down the opposite wall, a total of 42 feet. However, if you detach the sides and ceiling and lay them flat, as shown in the diagrams at the right, you see that the shortest distance from A to B is the straight line route of 40 feet.



13. George Washington was born on February 11, 1732 under the old style (Julian) calendar. In 1752 England and her colonies adopted the new style (Gregorian) calendar. Because of the change, September 1752 "lost" eleven days. In order to "make up" the "lost" days, George Washington began celebrating his birthday on February 22nd instead of February 11th.
14. a. 29 days b. 28 days Under the Gregorian calendar, leap year is omitted in the last year of each century unless the century is divisible by four (as in 20th century).
15. The nonagon is used in circlevision, (9 screens, 9 projectors) found in Epcot and other Disney parks and elsewhere.