A. Common Features of NSF–sponsored Curricula

1. The curricula are organized using multiple strands of algebraic, geometric, statistical, probabilistic, numerical and discrete mathematical ideas, which build upon each other throughout each grade level;
2. Core mathematical ideas within each strand are carefully sequenced and articulated with each other through more advanced grades;
3. These core ideas are conceptual integrated and presented in the form of thematic units or content strands designed to intrigue and engage students at different levels of depth and abstraction;
4. The curricula use modeling, group data collection, simulations and predictions;
5. Students work individually and in collaborative learning groups to actively investigate non-routine problems over an extended period of time;
6. Graphics and scientific calculators are used as an integral component of the lessons;
7. The curricula are college-preparatory material accessible to all students.

These design features of the above NSF-sponsored curricula, accompanied by student-centered teaching methods, are supported by a substantial body of cognitive science research (Bruer, 1993; Caine, R., & Caine, G. 1991; Piaget, 1971). These curricula presuppose students are inherently "active learners" who interpret and construct meaning from their engagement with interesting mathematical questions and concrete materials. In these curricula, a series of carefully sequenced activities lead students to discover relationships and, therefore, acquire deeper conceptual understanding of important mathematical ideas. Students work in small groups and collaborate on developing strategies to solve open-ended problems. Teachers guide the groups through "organized discovery" whereby the teacher asks a probing question provoking students to think rather than memorize. Students generate a variety of algorithms and are assessed using a variety of measures.

The internal organization of these new curricula is in sharp contrast to pre-standards texts, which organize and present mathematics formally, topic-by-topic, emphasizing algorithmic manipulations and computational tasks. A mathematics curriculum organized linearly by topic encourages an instructional method whereby the teacher stands and tells concepts and procedures to students, interspersed with teacher-led whole class questioning. Students in these classes typically sit passively in rows watching and listening as the teacher shows them a procedure on how to solve a particular problem. Students are then assigned homework problems to practice the day's new procedure and are later tested for mastery of the algorithms. The teacher then moves on to the next topic in an effort to "cover" the material. As a result, according to the *Third International Mathematics & Science Study (TIMSS)* (1997), the U.S. curriculum has become "a mile wide and an inch deep."

Interactive Mathematics Program-IMP

Level: 9-12

Developers: Dan Fendel, Diane Resek, Lynne Alper, Sherry Fraser

Publisher: Key Curriculum Press

Review Materials: Years 1 and 2 were reviewed in published form. Years 3 and 4 are complete and were reviewed. Some units are in final form and some are being field- tested. Teacher editions for each unit of each year were reviewed. Also reviewed and available from the publisher are the booklets: Introduction and Implementation Strategies for the Interactive Mathematics Program, Teaching Handbook for the Interactive Mathematics Program, and Guide to Using TI Calculators with IMP Year 1.

Format/Description: This is a complete four-year secondary school curriculum for all students. Each year consists of five units. Most units begin with the statement of a unit problem. Unit problems are long-term problems which usually require considerable mathematics to solve. Students' primary task is to explore and develop mathematics related to the problem in such a way that they will be able to produce a solution to the problem by the end of the unit. However, the mathematics developed in each unit extends beyond that necessary to solve that particular problem. The student units consist of m- class activities and daily homework. Most units include several Problems of the Week (POWs). The POWs are open-ended and may develop mathematics not directly related to the central unit problem. In addition, each unit contains supplemental problems. Some of these reinforce concepts or skills developed in the units; others are extensions of the basic classroom material and are included for students who are ready for additional challenges. The units integrate material from several areas of mathematics. For our purposes, we will define these areas, or threads, to be **algebra/number/function, geometry, trigonometry, probability and statistics, logic/reasoning, and discrete mathematics**. Not all of these threads are present in every unit and each receives varying coverage from year to year. There is a prescribed order to the units as later material development is based on prior work. Indeed, previous knowledge, often from a different thread, is skillfully integrated with developing knowledge. Moreover, many scenarios from previous units return with new twists. This helps connect new content with previous knowledge. It is possible for students to enter the curriculum at many points. However, since occasions where previous material is needed are often clearly designated in the teacher materials as places where review can occur, many times the needed material can be introduced at that point. Nonetheless, students who pass through the entire four years of the curriculum will reap the full benefits of the materials as they are written. The instructional materials promote deep understanding of concepts and students often utilize the experiment-conjecture-prove (or: work on concrete cases-generalize-explain) paradigm when solving problems. Mathematical thinking is paramount. Furthermore, the content is developed in such a way that the students see a need for the mathematics being developed as they progress, rather than just application of the content after it is presented. The curriculum involves extensive oral and written communication by students during all the activities, homework, and POWs. Some assignments ask students to summarize they have learned; other work includes a report of results as if, for example, they were professional consultants.

The teacher materials contain overviews of the units, a list of concepts and skills the students will be learning, a materials list, daily lesson frameworks (which can be modified), information concerning how concepts might evolve, suggestions concerning the in-class activities, and sample answers for the activities, homework, and most of the POWs. Blackline masters and assessment suggestions are also included. Recommendations on topics teachers might discuss with their peers are incorporated. For additional information, see the booklet Teaching Handbook for the Interactive Mathematics Program.

Pedagogy: The in-class activities are designed to be done collaboratively in groups. It is recommended to use groups of four when possible and that the groups be randomly assigned (see Teaching Handbook for the Interactive Mathematics Program). However, students usually work independently on homework and POWs. If students do not work individually, they have to specify their collaborators. Write-ups on POWs must be done individually, at least. Class discussion occurs often and groups of students, as well as individual students, present their results to the class routinely. As mentioned above, many forms of writing; including reports, summaries, free-focused exercises, letters, and daily homework, are stressed. Students are required to justify their results and explain their reasoning in writing and orally during presentations. **Indeed, when implemented as intended, it is the students who develop the mathematics, through guided discovery, rather than having the mathematics presented in a detached fashion by the teacher or the textbook.**

Technology: The curriculum incorporates the use of graphing calculators, although not all units require their use. However, it is expected that such calculators are available to students while they are in class and the students decide when to use them. They use the calculators to do tedious calculations, simulations, create mathematical models, and create graphics. In one unit they do some programming using the graphing calculator with the capabilities of a TI-82.

Assessment: In the words of the developers, "assessment should be part of the natural flow of the classroom." This quote and other ideas concerning student assessment appear in the booklet: Teaching Handbook for the Interactive Mathematics Program. There, teachers can find suggested approaches to assessment and grading built on homework assignments, POWs, oral presentations, write-ups of class activities, and end-of-unit assessments. It is anticipated that assessment of some sort take place every day. An overview section of each unit's teacher's guide suggests a specific selection of assignments along with POWs and end-of-unit assessments for the purpose of assigning grades.

Also, at the end of each unit, each student writes a cover letter reflecting on the mathematics in the unit. This letter, together with samples of unit work selected by the student, are assembled into a student portfolio. The collection of portfolios form a growing picture of student learning. These portfolios are not only instructive for students and teachers, but are also useful for parents, administrators, and, sometimes, college officials.

<u>Content Overview:</u> The first four NCTM Standards: Mathematics as Problem Solving, Mathematics as Communication, Mathematics as Reasoning, and Mathematical Connections are addressed heavily in every unit. Students are consistently asked thought-provoking questions and are required to explain their reasoning. As stated above, the students are constantly asked to communicate mathematically, both orally and in writing. Many problems, especially the POWs, are open-ended. The curriculum is integrated, using the threads algebra/number/function, geometry, trigonometry, probability and statistics, logic, and discrete mathematics as mentioned above. The curriculum is problem-centered. That is, each unit begins with central problem that is usually quite difficult and takes a considerable amount of mathematics to solve, usually mathematics from several of the threads in the current and previous units. The unit problems are not necessarily set in serious real world applications, but rather are set in contexts which have proved interesting and motivational for students to develop the mathematics necessary to solve the problems. These unit problems can be explored on many levels and will challenge the brightest students as well as provide a framework for any student to do meaningful mathematics. In addition, within the unit, there are smaller problems that provide a focus for developing the mathematics. The curriculum stresses in-depth understanding of concepts and techniques and ways to apply them.

The following describes each year according to the mathematical development within the threads mentioned above. For the most part, topics that are mentioned briefly and not emphasized are omitted. In some cases, however, we state that students have been exposed to a topic or are familiar with a topic if the emphasis is light, because units in later may revisit and expand on the light emphasis. Moreover, when topics could be placed under more than one thread, an often arbitrary decision is made to place the topic under one of the threads. The catalogue of content under a heading is not necessarily listed in the order it is developed during the year. **<u>In what follows, whenever it is said that a student will be able to do a task or has a strategy to solve a problem. it is implied that the student will also be able to explain, clearly, what is being done and why it is been done.</u>**
Moreover, throughout the curriculum students 'II use inductive reasoning to determine patterns or develop conjectures, and deductive reasoning to present proofs of many statements. While some proofs are rigorous, some are less rigorous and some are intuitive. However, students are constantly urged to give whatever explanations they can for results and these explanations are evaluated by the teachers as well as peers. Indeed, as the curriculum evolves, students are lead to an understanding that mathematical knowledge is much more than mathematical "coping" in order to get by for an examination order to pass; that mathematical knowledge includes not only the ability to use a concept or result, but also the knowledge of when to use it, why a result is true and why a concept or result was invented as well as the ability to communicate this understanding to others. Furthermore, students are guided to become independent learners. For example, by the middle of Year 4 students should be able to use traditional secondary mathematics textbooks to gain knowledge about topics other than those covered in this curriculum or topics where coverage is lighter than traditionally allotted.

Year I

Algebra/number/function: Students will be introduced to "In-Out' machines (function machines) and "In-Out" tables for one and two variables, which they "I be able to utilize to collect and organize data. They will discover geometric and numerical patterns in tables of data as well as in problems posed. They will be able to describe a pattern in words and, in many cases, create an algebraic expression using one or two variables to generalize this pattern. Techniques such as drawing a diagram/picture or using a specific case are encouraged to help clarify generalizations. They will explore some recursive functions (term "recursive" is optional) that involve triangular numbers and permutations (not stated as such, but factorial notation may be introduced); however, emphasis is clearly on the recognition of overall patterns. They will write a program for the graphing calculator that simulates a function machine. Students will be able to create an algebraic expression, using more than one variable, to represent a situation, and write a "summary phrase" (concise verbal description) to relate its meaning. They will be able to evaluate algebraic expressions with one or more variables. Given a "graph sketch" (an unscaled graph), they will be able to interpret and describe a real-life situation that could be represented by the sketch. [Both continuous and discrete functions are considered.] In addition, they will be able to sketch a graph given a description of a relationship between two variables. Students will be able to explain the connection between various situations and their representations (tables, graphs, and symbolic rules). They will be familiar with the notion of independent and dependent variables as they relate to the input and output of In-Out tables. They "I be able to construct a table of values from information depicted in a coordinatized linear graph and write a rule to represent the general relationship. Given a linear or quadratic equation, they will be able to make a table of values and draw a graph by plotting points. Given a table of values, they will be able to plot points, determine a line or curve (sometimes using the graphing calculator) that can best approximate the data points, find its equation, and make predictions based on that equation. Students will be introduced to the intuitive concept of slope as it relates to the steepness of a graph. They will be able to write a linear equation in "y=" form. Using a graphing calculator, they will be able to graph linear and quadratic functions and use the trace feature to evaluate the function at specific values. Students will be able to graph two linear equations on the same set of coordinate axes and determine a simultaneous (term is not used) solution by finding the point of intersection of the two lines. They will investigate the distance-rate-time relationship and be able to solve problems involving various kinds of rates (rate of travel, rate of water consumption, rate of profit). Given a problem, students will be able to identify variables that could describe a functional relationship. They will be introduced to function notation, including more than one variable (e.g. S=f(L,D,H)). Utilizing both experimental (table of data) and analytic (e.g. looking at the geometry of a problem) methods, they will determine an equation or formula for a given situation. They will be able to create proportions for similar triangle problems and solve them through primarily intuitive approaches such as trial and error. Cross-multiplication may be discussed and used, though students are expected to Justify any method they employ. They will become familiar with solving for one variable in terms of one or more other variables.

Students will be able to evaluate arithmetic expressions using the correct order of operations. They will become familiar with sigma notation (□) to indicate the sum of a finite number of integers. They will investigate patterns in sums of consecutive natural numbers and make and test conjectures, such as:

"Any odd number greater than I can be written as the sum of two consecutive numbers." Students will be able to add, subtract, and multiply integers, using a "hot-and-cold-cube' model (hot cubes represent positive integers, cold cubes represent negative integers), using a thermometer model, and by looking for a pattern in a sequence of arithmetic equations. Absolute value is introduced in terms of the number of cubes an integer represents.

Geometry: Students will be introduced to angles from two perspectives, i.e. dynamic, as turns (number of degrees in a rotation), and static, as geometric figures (e.g. ▲ABC). They will be able to find the measure of an angle of a polygon (triangle, hexagon, rhombus, square, trapezoid) by fitting pattern blocks together to determine the appropriate fractional part of 360°. They will be able to measure an angle using a protractor. They will explore patterns in sums of interior angles of polygons in terms of their sides and explain why an angle sum is (n-2)180° for an n-sided polygon, using the assumption that the sum of the angles of a triangle is 180°. [They return to this assumption in a later unit and prove it.] Students will understand the concept of similarity, both intuitively and as a formal definition. They will be able to identify characteristics of similar figures; in particular, they will explore the rigidity of triangles (as opposed to the lack of rigidity in other polygons) and know that two triangles are similar if two of their corresponding angles are equal or if their corresponding sides are proportional. They will investigate angle relationships (corresponding, supplementary, complementary, vertical, alternate interior), including those formed by a transversal intersecting two or more lines, and draw general conclusions (e.g. If parallel lines are cut by a transversal, corresponding angles are equal). They will recognize that a line drawn through a triangle and parallel to one side will produce a triangle similar to the original triangle. They will be able to create mathematical models involving similar triangles, including scale drawings, to indirectly measure the height or length of an object.

Trigonometry: Students will discover that the trigonometric ratios of right triangles are constant for any given acute angle. They will be introduced to notation (sin, cos, tan, cot, csc, sec) and encounter some relationships between these ratios (e.g. sinA=cos (90°-A)). Given a problem and its right triangle representation, they will use a trigonometric equation to find a missing length of a side. They will develop a general trigonometric relationship for a sun shadow problem (S=H/tan).

Probability and statistics: Students will be able to devise an experiment to estimate the probability of success using a specific strategy, and they will understand that accuracy of their estimate usually depends upon the number of times the experiment is repeated. They "I recognize that the probability of an event is a number between 0 and I inclusive and they will be able to determine the probabilities for events involving equally likely outcomes. They will be familiar with the concept of independent events and ascertain whether previous events affect future events in given situations. They will be able to find probabilities (including the probability of a sequence of events) by using and creating an area model that can be subdivided into regions which can represent equally likely events, by developing a tree diagram, or by listing and counting all possible outcomes. They will be able to distinguish between and compare experimental probabilities and theoretical probabilities. They will be able to calculate the expected value for various situations, including multi-stage (2 or 3) events, using the "large number of trials" method. They will be able to determine how to adjust payoffs so that a game is considered fair. Students will investigate a variety of strategies in the pursuit of a "best strategy" to play a game or make

a decision in real-life situations. They will be able to analyze and compare these strategies by drawing an area model and finding the expected value for each.
Students will be able to predict and compute a mean for a set of data. They will be able to construct frequency bar graphs, including those that group data into intervals along the horizontal axis. They will investigate measurement variation and normal distribution as they pursue their objective to design and conduct experiments that consider the effect of a given variable on an outcome. They will explore the dispersion of data by finding the range of variation for "ordinary" and "rare" events in a normal distribution, by comparing sets of data that have the same mean but a different spread, and by examining several methods to measure data spread. They will be able to calculate the standard deviation for a set of data, and compare and arrange data sets in terms of their spread.

Logic/reasoning: Students will be introduced, informally, to the concept of proof as a "completely convincing argument" within a larger context of reasoning. They will use logical reasoning to justify their solutions. They will become familiar with proof using cases. They will be able to find counterexamples to establish that a statement (in particular, an "if…then" statement) is false. They will be able to formulate and test conjectures for a variety of situations, such as the sum of the interior angles of a polygon and sums of consecutive positive integers.

Discrete Mathematics: See Algebra/number/function above.

Year 2

Algebra/number/function: Students will enhance their ability to represent real-life situations in terms of equations, tables, and graphs, and they will understand the connections between these representations. Given a problem, they will be able to define appropriate variables and write an equation. They will create their own stories to fit a given equation. Using a pan balance model, they will explore strategies for solving equations in one variable and be able to justify each step they take in their procedure. They will be able to check a solution by substituting into the original equation and evaluating the result. By constructing an area model, students will investigate multiplication of algebraic expressions (including the product of two binomials) and "discover" the distributive property. They will be able to use the distributive property to multiply algebraic expressions and to find common factors.
They will be able to find a product of algebraic sums using an area model or the multidigit multiplication algorithm. They will be able to simplify algebraic expressions and solve linear equations that contain parentheses. Using equivalent equations, they will be able to solve linear equations in one variable.
They will be able to solve equations for one variable in terms of another variable(s). Using functional notation, students will be able to evaluate a function at a particular value. They will be able to create a table of values and graph a linear function. They will explore the relationship between a linear function and its graph, including the effect of the coefficient of the "x" term. Given a general formula (e.g. m(t)=O.1t^2+3t), they will create an equation for a specific value (e.g. 0.1t^2+3t=200) and solve the equation by examining the graph of the original function. They will be able to analyze and interpret a graph of a function whose rule is not given. Students will be able to use equivalent inequalities to solve a linear inequality in one variable, as well as simplify a linear inequality in two variables. They will be

able to graph a linear inequality, in one or two variables. They will investigate methods (including trial and error and graphing) for solving systems of linear equations in two variables. Furthermore, they will develop and use at least one algebraic algorithm, and describe the advantages and disadvantages of graphical and algebraic methods for solving systems of equations. They will create their own "two equation/two unknown" word problems, along with written solutions. Given a linear programming problem students will be able to identify appropriate variables, generate a set of inequalities to represent the constraints on the situation, write a linear expression to be maximized or minimized, graph the feasible region, solve the problem and verify that the solution is the best within the constraints of the problem. In addition, they will create their own linear programming problem, providing a solution and proof that it is optimal. Given a real-world situation involving exponential growth or decay, students will construct a graph of data points and find an exponential rule that can represent the graph. They will graph y=2^x and y=x^2 on the same set of axes and compare the growth of each function. By examining the classic chessboard problem (one coin placed on the first square, two on the second, four on the third, etc.), they will recognize the power of exponential growth. They mill be familiar with solving exponential equations by trial and error (e.g. 2'=10), and be introduced to the logarithm as the solution to an exponential equation (x=loga,b). They will sketch and compare two logarithm functions with different bases on the same set of axes. They will also compare the graph of a logarithm function with its corresponding exponential function.

Students will review prime and composite numbers, and be able to write the prime factorization of a number. They will explore the number of divisors any number has, formulate questions, and make conjectures based on their observations (e.g. "What kinds of numbers have exactly three divisors?"). Students will examine and apply properties of the square root function, in particular √a. √b=√(a.b) and √(a/b)=√a/√b. They will investigate the effect of rounding off early on in a computation to subsequent calculation results. Students will develop and use rules of exponents [a^x.a^y=a^x+y , (a^x)^y=a^x.y , a^x .b^x=(a.b)^x and a^(p/q)=(q√a)^p] They will explore several different ways to make sense of the definitions $a^0=1$, $a^1=a$, a^(-x)=1/a^x. They will investigate the definition of scientific notation and be able to convert ordinary numbers to scientific notation and vice versa. They will develop some general principles for multiplying and dividing numbers and use these to solve real-world problems involving scientific notation.

Geometry: Students will utilize geoboards to investigate the concept of area, including unit of measurement. They will be able to approximate the area of a figure using a nonstandard unit of measure. They will be able create figures (including triangles and quadrilaterals) on a geoboard and find their areas. They will develop formulas for the area of a triangle, parallelogram, and trapezoid. They will realize that figures with the same perimeter can have different areas. By examining relationships among the areas of squares made from sides of a triangle, students will "discover" the Pythagorean Theorem, explore a geometric proof of it, and be able to apply the theorem to problem situations. They will also investigate the triangle inequality. Given the lengths of the sides of any triangle, they "I be able to find the area. They will ascertain that a square has maximum area of all rectangles with a fixed perimeter. They will explore and compute areas of regular polygons with fixed perimeters and conclude that the greater the number of sides, the larger the area. They will recognize that areas of similar polygons are proportional to the squares of their perimeters. They will develop a formula to find the area of an n-gon with fixed perimeter. Students will be able to create a two-dimensional net for a three-dimensional rectangular solid. They will develop formulas for the surface area and volume of a rectangular solid, and recognize that figures with the same volume can have different surface areas. They will develop and use formulas for the lateral surface area and volume of a night prism, and recognize that figures with the same lateral surface area can have different volumes. They will generalize the Pythagorean Theorem to three dimensions. They will establish the general principles that volumes of similar solids are proportional to the cubes of their corresponding linear dimensions, and that surface areas are proportional to the squares of their perimeters. Students will investigate tesselations of regular polygons and discover that only equilateral triangles, squares, and regular hexagons tesselate.

Trigonometry: Students will review right triangle trigonometry in terms of ratios of corresponding sides of singular triangles. They will be able to compute the sine, cosine, and tangent of an acute angle by measuring the sides of a right triangle and forming the appropriate trigonometric ratio. They will strengthen their ability to solve problems involving right triangle representations by setting up and solving a trigonometric equation to find a missing length of a side. Given one angle and the length of one side of any triangle, they will be able to use trigonometric ratios to find the height of the triangle. Students will begin to investigate some general characteristics of sine, cosine, and tangent functions, including the range of each. Using the calculator, they will explore the inverse trigonometric functions (tan-', sin-', Cos-') to find the measure of an angle.

Probability and statistics: Students will enhance their ability to conduct experiments in order to estimate probabilities. They will analyze a problem involving conditional probabilities (term is not used).

Given a real-world situation, students will be able to identify an appropriate population and a representative sample of the population. They will recognize that observed differences in samples could be due to sampling fluctuations. They will be able to construct a double bar graph for an applicable set of data. They will be able to formulate hypotheses based on actual data. When comparing a population with a theoretical model or another "real" population, students will be able to devise a hypothesis and a null hypothesis, design and carry out a plan for gathering data (if necessary), and analyze results by calculating the - ' statistic and interpreting it using the - ' probability chart and distribution curve. Based on this analysis, they will be able to decide whether or not the two populations are really different, i.e. whether or not to reject the null hypothesis. They will use proportional reasoning to find expected numbers when assuming a true null hypothesis in comparing two "real" populations. They will become familiar with the distinction between correlation and causation, i.e. a correlation between two variables does not necessarily indicate that one caused the other. Students will review normal distribution and standard deviation, use the standard deviation of a normal distribution to draw conclusions about hypotheses made, and compare the statistic with standard deviation.

Logic/reasoning: Students will continue to develop their reasoning skills; in particular, the POWs are more generally phrased and provide a framework for forrnwating "completely convincing arguments." They will be expected to provide full explanations of solutions and, when appropriate, a proof that a solution is correct. Students will be familiar with analyzing a situation on a case by case basis in order to conclude that there is a unique solution. They will use deductive reasoning to draw valid conclusions from two or more statements assumed to be true.

Discrete Mathematics: See Algebra/number/function above.

Year 3

Algebra/number/function: Students mill strengthen their ability to determine the variables, the relationships between them, and make simplifying assumptions in situations described verbally (and in writing). They will also strengthen their ability to use such mathematical models, particularly many situations where their models involve linear, quadratic, or exponential functions, to predict behavior of the variables in contextual situations. They will strengthen their ability to work with algebraic symbols. They will be able to re-write quadratic functions and equations given in general the form, $y = ax^2+bx +c$ using forms more useful for analysis as long as a, b, and c are specific numbers (rather than parameters). They will understand the connection between the coefficients of quadratic functions and the geometry of the graph. That is, they will know what changes each of coefficients a, b and c do to the graph of the function. They will be able to factor simple quadratics with integer coefficients where the coefficient of x^2 is 1. They will be able to use such a "factored form" of a quadratic function to determine (or verify) the x-intercepts of the graph of the function. They will be able to explain how and why this factored form delivers the roots of the quadratic using the "zero" property of multiplication (the product of two numbers is zero only if at least one of the numbers is zero). They will have strategies for determining when such quadratics with integer coefficients (and the coefficient of x^2 is 1, i.e monic polynomials) are not "factorable" (at least don't have linear factors with integer coefficients).
Conversely, they will be able to multiply factored quadratics to get the general form of the function and be able to connect such multiplication to area diagrams of squares where each side has length equal to one of the factors. By completing the square, students will be able to re-write a quadratic function, usually a monic polynomial, in "vertex form:" $a(x+h)^2+k$ as long as h and k are specific numbers (and usually where a=1). They may or may not be able to convert the general function $y = ax^2+bx=c$ to vertex form but they will understand that any quadratic (with specific numbers as coefficients) can be converted to vertex form. They will be able to use vertex form to give the vertex or "turning point" for the function and be able to prove why this point is the turning point by using algebraic properties of the vertex form, such as $(x+h)^2$ is always positive. (They will have other strategies for estimating the turning point such as graphing or trial and error.) Similarly, they will be able to use the algebraic properties of the vertex form to explain why all quadratic functions have graphs with same general shape. They will know what changes in the parameters a, h, and k do to the graph. They will know that the a in the vertex form is the lead coefficient of the quadratic function and that the sign of this number determines whether the graph of the function opens up or opens down. They will be exposed to finding roots of a quadratic using the vertex form in specific cases, but will probably not know the quadratic formula at this time. They will be able to use quadratic functions to create mathematical models and do mathematical analyses in a variety of settings. For example, using the algebraic properties of the vertex representation of a quadratic they will be able to prove that of all rectangles with a perimeter of 200 feet, a square is the rectangle with the largest area. In addition, they will be able to fit a quadratic to three "data" points in the coordinate plane, where the graph contains each of the points. Students will

know and be able to use such standard terminology as quadratic, parabola, polynomial, trinomial, coefficient, term, maximum (minimum) of a function.

Students will be able to solve systems of equations in two unknowns and systems of equations in three unknowns using both substitution and elimination methods, as well as matrix methods discussed in the **discrete mathematics** section below. (They will also be able to solve a system with two unknowns graphically and, occasionally, both two and three-dimensional systems using common sense trial-and-error.) As an example, they will be able to fit a linear function to two data points using the coefficients (parameters) as variables. They will also be able to fit a exponential functions to two data points. Students will understand and be able to describe inconsistent and dependent systems in terms of the system solutions (no solutions and infinitely many solutions, respectively.) They will be able to connect the algebraic and geometric descriptions of inconsistent and dependent systems in systems of two or three dimensions. Students will know what a Pythagorean Triple is and be able to prove that if each number in a Pythagorean triple is multiplied by the same (positive) constant, a new Pythagorean Triple is formed. Students "I understand and be able to compute the average rate of change of a function as the x-coordinate moves from one given point to another, particularly in the case where the function is a distance function and the horizontal coordinate is time. They will understand the derivative in this example as the instantaneous velocity at a point in time and be able to generalize this to the instantaneous rate of growth of a function. They "I know that in order to determine the value of the derivative one must investigate what happens to average rates of change where the change in the horizontal coordinate is determined by a short interval beginning shortly before or after the point in question. They will be able to "determine" derivatives by making numerical computations, from formulas or graphs, of average rates of change as the horizontal change shrinks; especially for linear, quadratic, and exponential functions. They will know and be able to explain that the derivative of a linear function at any point is the slope of the line determined by its graph and interpret that as the "absolute growth rate" of the function in this case. They will know and be able to explain why, although not rigorously, the derivative of an exponential function y = (kb)^cx is proportional to value of the function at that point and interpret this as "constant relative growth." They will also know (but will not have seen a formal proof) that the class of exponential function is characterized by constant relative growth. They will have an intuitive understanding of the concept of limit as it pertains to the difference quotient in the determination of the derivative. (The term difference quotient is not used.) Students will be able to change bases in exponential expressions. That is, given two positive numbers a and b, using logarithms, students will be able to find a positive number c such that a^x= b^cx, for 0 (real) values of x. They will understand Euler's constant, e in two ways. First, as the intuitive "limit" of the expression (1 +1/n)^1/n as the positive integer n gets larger and larger. Second, as the base of an exponential y = e^x where the quotient of the value of the function and the value of the derivative is 1. They will also know the terms (and be able to use) common and natural logarithms. They also will have developed and know the closed form formula for sum of n terms in an arithmetic sequence.

Geometry: Students will understand the concept of slope of a line geometrically as the rise over the run as one moves toward the right. They will be able to associate the sign of the slope with the direction that the line tilts. They will know that horizontal lines have zero slope and they "' I be able to explain why slope is undefined for vertical lines. They will also know the slope as the ratio of the

difference of the y-coordinates to the x-coordinates. They will be able to determine the slope of a line given two points on the line and understand that the computation is independent of the points chosen as a result of the geometric similarity of triangles determined by the line and the two points. They will be able to connect the geometric properties of y-intercept and slope of a line to the coefficients in the algebraic representation of the line written as $y=a+bx$. They will be able to determine the formula for a linear function given two points on the graph or one point on the graph and the slope. They will also be able to represent the slope of a line as a rate of growth in many contexts. Students will understand the basic shape of an exponential function y=(kb)^cx
, for O<b. They will be able to compare linear and exponential functions. For example, in the expressions above, changes in the linear parameter a and the exponential parameter k both change the y-intercept. Changes in the linear parameter b and the exponential parameter c both change the rate of growth of the function. Students will understand the derivative as the instantaneous rate of change, or growth, of the function at a point (x-value) and that it can, except in the case of a linear function, be depicted geometrically as the slope of the line tangent to a curve determined by the graph of a function over that point. They "I understand that a positive derivative indicates that the function is increasing at that point, a negative derivative indicates that the function is decreasing at that point and that at "turning points" the derivative is zero, if it exists. They will also have explored at least one case where a function does not have a derivative at a point. They will know that the magnitude of the absolute value of the derivative of a function at a point determines its steepness. They will be able to interpret the derivative of a function as a new function in some instances. Students will be able to write (and will have rigorously developed) the algebraic equation for the equation of a circle with specified center and radius in the coordinate (Cartesian) plane. Moreover, they will be able to transform the general equation $x^2+bx+y^2+dy+e = 0$ into the form $(x-a)^2 +(y-c)^2 = r^2$, when b, d, and e are specific numbers. (There is even some exposure to imaginary numbers when r is negative.) They will be able to use (and will have rigorously developed) the distance formula between two points. They will be able to find the distance between a point and a line (not containing the point) in coordinate geometry, although no general formula is developed. They Will know and be able to use (and at least experimentally developed) the formula for the midpoint of a line segment in coordinate geometry. Using results and techniques in synthetic geometry, coordinate geometry and trigonometry, students will develop strategies to prove or establish (at least with a partial proof) many results from Euclidean geometry, such as (but not limited to): a point equidistant from two points is on the perpendicular bisector of the line segment joining the two points; a line through the midpoint of a segment is equidistant from the two end points of the segment; there is a point equidistant from three
if and only if the three points are not collinear; every triangle has an inscribed circle; vertical angles are equal; the bisectors of each pair of vertical angles formed by two intersecting lines are perpendicular; the tangent to a circle is perpendicular to the radius of the circle. (Their strategies for proof should extend beyond the particular results they prove.) Students will have deepened their insight (or developed insight) into formulas for the area and circumference of circles by developing an outline for a proof (some details are omitted) that the circumference of a circle of radius r is $2r\pi$ and the area is πr^2. They will have strategies for estimating the value of pi. They will have developed and be able to use the formula for the volume and surface area of a cylinder given the radius of the base and its height. They will know and be able to use such terms as cross-sectional area, equidistant, circumscribed/inscribed circle/polygon, angle bisector, perpendicular bisector of a segment, line, tangent to a circle, (integer) lattice point in the coordinate plane, and unit circle. Students will deepen

their ability to solve two-dimensional linear programming problems graphically and be able to describe geometric properties of the feasible region.

Students will know and be able to use the standard coordinatization and associated visual representation of three-space. They will know the connection between linear equations in three variables and their graphical representation as planes in three dimensions. They will understand the geometry of lines and planes in three dimensions. For example, they will be able to describe the possibilities for intersections of two, three, and four planes in three-space (resulting in a plane, a line, a point, or the empty set.) They will deepen their understanding of parallel lines in three dimensions as more than lines that do not meet. That is they will understand the difference between parallel lines (which must be in a plane) and skew lines (which don't meet, but are not in a single plane).

Trigonometry: Students use their knowledge of right triangle trigonometry in geometric situations. See **geometry** above.

Probability and statistics: Students will enhance their ability to use area and tree diagrams (some with probabilities as branch weights) and simulation to analyze sequences of events. They "I focus on three phases in the mathematical process: knowing how to compute, knowing when to apply, and knowing why a method works. They mill be able determine the number of outcomes for a multi-stage event using several strategies: organized lists (when the number is small), the multiplication principle, tree diagrams, and identifying situations that involve permutations or combinations. They will understand the multiplication principle as the number of ways one can chose one object from each of two disjoint sets and its generalization to choosing one object from each of n disjoint sets. They will be able to justify why one multiplies the probability weights along the branches to an outcome in order to establish the probability of the outcome. They will recognize that the concept of permutations may be utilized in situations where multi-stage outcomes are different if the sequencing of the individual stages, and their respective outcomes, is changed. They will recognize that combinations are useful in situations where outcomes are not considered different if only the sequencing of the stages is changed. They will know and be able to use factorials. They will know, be able to use, and have developed symbolic formulas for nPr, the number of permutations of r objects selected from n objects (n,r,0). They will know, be able to use and have developed symbolic formulas for nCr, the number of combinations of r objects selected form n objects (n,r,0). They will be able to calculate both nPr, and nCr, using pencil and paper if r is small and using technology otherwise. They will be able to use these concepts in situations involving equally likely probabilities. They will also be able to use combinatorial coefficients in situations where outcomes are not equally likely. For instance, they will understand the term binomial experiment as a situation involving just two outcomes, *success* and *failure*, repeated a number of times. They will be able to determine when binomial probabilities are appropriate in situations and be able to utilize and calculate binomial probabilities or a cumulative distribution of binomial probabilities in situations that prescribe both the number of trials and the probability of success. They will recognize the general shape of a binomial distribution displayed in a bar graph and know, for instance, that the distribution is symmetric when the probability of success is 0.5. They will have strategies for determining the "most probable" outcome in some situations. They will be able to recognize Pascal's Triangle and know several patterns in it. For example, they will know that each row (other than the first

row, which only has one entry in it) begins and ends with a I and that each internal entry has the property that its value is the sum of the two entries in the previous row closest to the given entry. They will also know that the sum of entries in a row is a power of 2 (and be able to justify this with a combinatorial proof). They will know and understand the relation of the entries of Pascal's Triangle to the combinatorial coefficients, ${}_nC_r$, and the coefficients in the expansion of (a + b)^n. They will know ${}_1C_r$
and be able to justify several identities involving the combinatorial coefficients such as ${}_nC_r = {}_{n-1}C_{r-1},{}_{n-1}C_r$; ${}_nC_r = {}_nC_{n-r}$; ${}_nC_o = {}_nC_n = 1$; and
${}_nC_r = {}_nP_r /r!$. Students will know and be able to use the strategy of determining the probability of the complementary event as a way calculating the probability of an event. They will enhance their ability to use probability to evaluate null hypotheses and make decisions. They will also be able to use the pigeon hole principle in counting arguments.

Logic/reasoning: Students will know and be able to formulate the converse of an *if…then* (conditional) statement; particularly in geometry. They will also know the meaning of if and only if and be able to write two conditional statements given an if and only if statement in geometry. Students will have developed strategies for direct proof of many theorems in Euclidean geometry. They will develop some conversational proofs for some facts in linear algebra, such as: if a square coefficient matrix has an inverse, then the corresponding system of linear equations has a unique solution. The will also work on mathematical reasoning in terms of situations such as winning strategies for games or solutions to card tricks. They will enhance their ability to make simplifying assumptions in modeling situations. They will develop the strategy of making analogies, such as relating counting situations to either ice cream cone
possibilities (permutations) or bowls of ice cream (combinations). They 'II justify several combinatorial identities on the abstract level such as
${}_nC_r = {}_nC_{n-r}$ or ${}_nC_i =$ 2^n .

Discrete Mathematics: Students will deepen their understanding of linear programming problems (Ipp's). They will review and extend their knowledge of two dimensional lpp's. They will be able to describe the general (mathematical) form of an lpp in n variables using constraints that contain both equalities and inequalities. (However, terms such as profit function or cost function are used instead of objective function.) They will understand the geometry of the three dimensional case. They will be able to model verbal (written) situations with Ipp's in many cases and using up to six variables. They will be able to use matrices to represent many types of information. In particular, they will be able to use a matrix equation of the form [Al [X]=[B] to describe a given system of linear equations in n unknowns where [Al is the matrix of coefficients and [B] is the constant matrix. They will have actually worked with as many as six unknowns. They will have developed the operations of addition and multiplication of matrices (and know when multiplication of matrices is possible) and apply these operations in many contexts. They will have developed several properties of matrix algebra. They will know that multiplication of matrices is associative (when it is defined) and is not commutative, in general. They will have developed the identity matrix for multiplication of square matrices and know that some matrices have inverses. They will be able to find the inverse of some two dimensional square matrices using strategies for solving systems of equations in two unknowns and be able to use technology (graphing calculators) to find inverses of matrices (when they exist) for square matrices of larger dimensions (where dimension limitations depend upon the technology). They will know the relationship between being able to solve a system n linear equations in n unknowns and the existence of the inverse of the coefficient matrix, [Al. They will be able to use the inverse of the coefficient matrix (when it exists) to a system of n linear equations in n unknowns They will have done this cases when n=6.

Students will know the "corner point theorem" in the theory of linear programming for lpp's in n unknowns. (That is, if an lpp has a solution, it occurs at some corner of the feasible set.) From their visualizations in two and three dimensions, they will believe that this theorem is reasonable. (They do not see a formal proof.) They will have developed an algorithm for finding the solution to an lpp, when it has a solution, using their strategies for solving systems of linear equations. In the process, they should have developed strategies for listing all combinations of r objects taken from n objects for relatively large values of n and r, such as n = 12 and r = 6. (Students will not have encountered the formula for binomial coefficients in this curriculum at this point.) Moreover, they will have been exposed to the complications that arise if the "solutions" must be integer-valued. Students will be able to articulate why the matrix methods are efficient for more than two variables.

Year 4

Algebra/number/function: Students will extend their ability to determine variables and determine relationships between them in contextual problems. Many expressions and models they will develop will include the trigonometric functions sine and cosine. They will be able to develop an equation for distance traveled by an object with constant acceleration and a given initial velocity. They will understand four methods for solving algebraic equations: trial-and-error, symbol manipulation, by graphing, and through the use of formulas. For example, they will be able to assemble several formulas into a complex formula relating distance and time to describe motion of a Ferris wheel and be able to solve the resulting expression graphically using graphing calculator technology. Moreover, they will be able to work with the symbolic quadratic formula for general quadratic equations of the form $ax^2+bx+c = 0$ to solve for real roots of quadratic equations. (They will also have seen and worked through details of a derivation of the quadratic formula.) They will enhance their ability to develop quadratic equations and functions from contexts. Students will understand what complex numbers are and be able to perform the operations of addition, subtraction, and multiplication with complex numbers.

Students will know the formal definition of functions as a set of ordered pair in which no two ordered pair have the same first coordinate. They will also understand usual functional notation (such as f(x)=2x). They will be able to represent functions using tables, graphs, symbols (algebra or formulas), and verbal descriptions. They will be able to move between these representations in many instances. They will have considerable familiarity with several abstract families of functions such as: linear (y=ax+b), quadratic ($y=ax^2+bx+c$), cubic ($y=ax^3+bx^2+cx+d$) polynomial ($y=ax^n+bx^{n-1}+...+c$), exponential ($y=ab^x$),
sine (y=Asin(Bx)), reciprocal (y=K/x), rational (quotient of two polynomials), and, to a lesser degree, power functions ($y=ax^p$). Less formally, they will understand step functions and absolute value functions. They will know the basic shapes of graphs of functions in each of these families, except, probably, the general polynomial, general rational, and general power families. They will know that the linear family and the exponential family each as "two parameter" families and have strategies to "fit" unique functions from each o these two families to two data points. Similarly, they will know that the quadratic family is a three-parameter family and have strategies to fit quadratic function to three data points. They will be able to recognize patterns in tables that suggest linear,

quadratic, exponential, and sine. For example, they will know and be able to prove that linear functions have constant first differences, quadratics have constant second differences, exponential have constant ratios, and sine function tables portray periodic behavior. They will be able to determine by reasoning or using data which family of functions to use and which member of the family is most appropriate in many modeling situations. They will understand the concepts of domain of a function as the set of inputs or, implicitly, as the set of numbers for which a formula representing a function makes sense.
They will understand the range of a function as the set of outputs, or the set of values on gets from the inputs. They will also understand the terms dependent variable and independent variable. They will be able to determine functions as graphs that pass the "vertical line test." They will be able to distinguish discrete and continuous graphs and determine which would be more appropriate to model certain modeling situations. They will understand and be able to use the terms "proportional to" and "inversely proportional to" and determine the constant of proportionality in many modeling situations. Students will intuitively understand the concept of an asymptote as a line that a graph gets close to. They will have strategies to determine vertical and horizontal asymptotes in many situations involving rational functions and determine functions that have given asymptotes in other instances. They will know the formal symbolic definition of the sum, difference, product, quotient, and composition of two or more functions and be able to use these concepts in some modeling situations. They will also understand and be able to use the scalar product of a function. They will have strategies to determine the graphs of the sum and difference of two functions given graphs of the functions. They will be able to use composition in some modeling situations and they will know and be able to show cases where composition is not commutative. They will know the definition of an inverse function (and the special inverse sine function) and be able determine some function inverses from various functional representations (tables, graphs, and symbols), when they exist. They will understand the concept of an identity function and how and identity behaves in a composition with another function. They will know and be able to show that not all functions have an inverse. They will be able to describe the effects of changes in the value b in y=bf(x), y=f(bx), y=f(x)+b, and y=f(x+b) as it relates to changes in the graph of a function. They will be familiar with iteration of functions. They will understand the concept of a fixed point and have strategies to determine fixed points for some functions. They will understand the concept of fitting a graph to many data points in terms of regression (e.g. linear, quadratic, and exponential). They will understand that there is a measure of best fit (based on least squares) using representatives from a family of functions and that there is a measure of the quality of the fit (using the correlation coefficient). They will be able to use technology to find functions of best fit and their correlation coefficients.
Geometry: Using their knowledge of similar triangles, students will develop and be able to use formulas that give the coordinates of a point a given fraction of the distance along a line segment in terms of the endpoints of the segment in two and three dimensions. They will understand the basic geometric transformations (isometrics) of translation, rotation and reflection in two and three- dimensional coordinate geometry. (In this material, 3-space is often represented as an "extension" of the standard x y - plane with the z-axis pointing "out.") For example, they will understand how the coordinates of a point a-re affected by a translation in two and three dimensions, a reflection about an axis in two and three dimensions, a rotation about the origin in 2-space and a rotation about an axis in 3-space. (E.g. (x,y)•(x+ay+b) for a translation in 2-space). They will also have developed and be able to use matrix representations and matrix multiplication for rotations and reflections in order to

move a single point at a time or a finite set of points at a time ' two and three dimensions. They 'II be able to develop and use coordinate transformations to project (from "center of projection" or "viewpoint") points onto a plane in 3-space and use this technique to project a three dimensional cube onto its two dimensional image on a plane. Using their knowledge of coordinate geometry, geometric transformations and their matrix representations, they will develop computer algorithms and coded programs to animate the movement of three dimensional objects on a calculator (computer) screen. They will also know and be able to use the fact that the area of a triangle is one-half the product of the length of two of its sides and the sine of the angle of the triangle formed by the two sides. They will know and be able to use the algebraic expression for an ellipse in coordinate geometry.

Trigonometry: Students will be able to describe sine and cosine (developed previously in terms of right triangles) in terms of circular functions and any (positive or negative) angle degree measure. They 'II know the graphs of y=Asin(Bx) and y=Acos(Bx), for different values of A and B, and understand how the change in value of the parameters A and B effect the graphs. They will recognize the periodic behavior of these functions. They will be able to use sine and cosine to describe failing objects with an initial velocity in a given direction using the vertical and horizontal components of velocity. They will understand the inverse functions sin-'x and cos-'x (in terms of angle values) and understand to domain and range restrictions on these functions. They will have developed and be able to use several trigonometric identities including the Pythagorean identity $\sin^2 x + \cos^2 x = 1$; as well as sin(-°) = -sin(°), cos(-°) = cos(°); cos (°) = sin(90°- °); and sin(°) = sin(180°-°). In addition, they will be able to use the Law of Sines and the Law of Cosines. For example, given the measure of two angles and a side of a triangle, they will be able to determine the measure of the remaining sides and angles using the Law of Sines. They will know how to convert rectangular coordinates to polar coordinates. Students will also understand the meaning of radian measure of an angle and be familiar with the procedures for converting from one angle measure to another.

Probability and statistics: Students will enhance their understanding and ability to use the concepts of expected value, mean, standard deviation and the normal curve, particularly in terms of polls. They will develop a strong understanding of the role of standard deviation as it describes area under the curve. They will understand the process of random sampling. They will understand how sample size affects variation in sample results. They will understand the difference between sampling with replacement and without replacement. They will understand that if the population is large relative to the sample size, the difference between probabilities based on samples with replacement and without replacement is negligible. They will be able to determine and display probability distributions associated with samples (of fixed size) measuring a two-valued population characteristic. They will understand the concept of true percentage as the actual measure of the characteristic in the population and the sample percentage as the measure of the characteristic in the sample. They will know how to use the binomial distribution as a probabilistic model for the distribution of samples percentages for samples of fixed size in situations where one can ignore the difference between samples taken with and without replacement. They will understand how the concept of standard deviation is related to variation among the samples and how variation among samples is related to the concept of standard deviation. They will understand the Central Limit theorem as a statement which says that as the sample size increases, the distribution of sample percentages looks more and more like a normal distribution.

Students will have strategies to determine the mean and standard deviation of a probability distribution. They will have seen a relation between the probabilistic concepts of mean (expected value) and standard deviation and the corresponding statistical measures. Moreover, they will know and have used patterns to determine general formulas for the mean (expected value) and standard deviation of binomial distributions. They will also have such formulas for the percentage binomial distribution (when the x-axis represents the percentage of success in N trials rather than the number of trials). Students will also understand the term variance. They will be able to analyze reliability of samples from population where the true percentage and standard deviation are known. Conversely, for sample percentages which are assumed to be normally distributed, they will be able to use a single sample percentage, the standard deviation of the sample, and a strategy to determine an upper bound for the standard deviation of the distribution of sample percentages to analyze the reliability of the sample and to estimate the true percentage of the population characteristic. In particular, they will be able to *construct confidence intervals and determine margin of error.* They will also understand the terms *confidence level* and *confidence limits*. Moreover, they will understand the "popular" use of the term margin of error. In addition, given some information about confidence intervals (e.g. confidence level and margin of error) they will be able to determine sample size. Students "I also be able to use tables for the normal curve to derive conclusions from or about sample or population statistics. They will also have seen the general analytic function whose graph is the normal curve. They will know that the overall size of the population might not affect the reliability of the sample, provided the population is large enough for the sample size and is also large enough so that distinctions between sampling, with and without replacement can be ignored.

Logic/reasoning: Students will understand and be able to use the logic of loops, particularly the "for" loop using "counters" and step values, and nested loops in a computer program (for a programmable graphing calculator). They "I be able to read and create a structured program and program outlines in several contexts which involve variables that have to be initialized. They will also have experienced working with a technical manual. They will use inductive reasoning involving patterns to make conjectures concerning patterns. They will be able to prove some of these conjectures. As they have throughout the curriculum, they will continue to develop the approach or ability to analyze multi-parameter situations by varying one of the parameters at a time.

Discrete Mathematics: They will develop some recursive and closed form formulas from contextual situations.

C. Elements of Systemic Change

The promulgation of new mathematics content, teaching and assessment *standards* has coincided with the need to change the "system" at the state and school district levels. These systemic change efforts include:

- Establishing statewide curricular standards as a *matter of policy*;
- Reformulating student assessments in light of these new content standards;
- Aligning school and state administrative policies to support these standards;
- Arranging for intensive and extended whole-staff teacher professional development;
- Adopting standards-based curriculum.

The National Science Foundation has been the lead agency in supporting large-scale mathematics and science systemic reform. Since 1990 the NSF has launched various systemic programs: the *Statewide Systemic Initiative (SSIs)*, the *Rural Systemic Initiatives (RSIs)*, the *Urban Systemic Initiatives (USIs), and most recently the Local Systemic Initiatives (LSCs)* programs.

Many schools have realized they must re-train their mathematics faculty if these standards are to have any impact in the classroom. Teachers will need to learn new inquiry-based curricula and student-centered pedagogical techniques. They will need to infuse their classrooms with more statistics and probability, more algebraic and geometric problem solving, and more real-world applications. Such re-training requires facilitating a major paradigm shift in the habits of mind and behavior of traditionally schooled teachers. Even eager and willing teachers have difficulty making this transition. This is not an easy or quick task.

D. The Greater Philadelphia Secondary Mathematics Project

The Greater Philadelphia Secondary Mathematics Project (GPSMP) is a "Local Systemic Change" program funded by the National Science Foundation. It builds upon nearly five years of work in Philadelphia implementing the Interactive Mathematics Program. Its five-year mission is to include more NSF reform curricula for both middle and high schools, and extend its service to include schools and school districts in southeastern Pennsylvania and southern New Jersey. From a total of possible 1,100 schools in both states, this project will eventually work with approximately 20 school districts that have demonstrated readiness to engage in standards-based mathematics reform. The goal is to train **600** teachers over five years in this local area to who are able to sustain the reform process after this grant ends.

GPSMP Professional Services

The Greater Philadelphia Secondary Mathematics Project provides the necessary professional development, in-classroom follow-up mentoring, and administrative technical assistance to schools that have made a commitment to align the *curriculum* and *instructional practices* of their secondary mathematics classrooms with the new mathematics standards. A school district's professional development plan is developed in joint consultations with the GPSMP project directors and the district's <u>administrators</u> and <u>teachers</u>. All plans have the following components:

1) a multi-year, intensive in-service schedule,

2) student-centered instructional methods,

3) inquiry-based, integrated curriculum,

3) teacher leadership development,

4) provisions for classroom mentoring, and

5) administrative technical assistance.

There is no charge to participating school districts for the following services:

1. In-service Curriculum: The professional development plan for each districts' math staff is designed to meet the needs of a range of teachers who are at different stages in their careers, who have different teaching assignments in their schools, and who have had different amounts of previous professional development. The professional in-service curriculum consists of two types of training. Each teacher will have a somewhat different mix and schedule of training depending on the particular needs of the school and the teaching assignments and experience of its teachers. The first type is centered on one of several *NSF-sponsore*d middle and/or high school curricula, which have been selected. This type of training ranges from 180 to 240 hours depending on whether the curriculum spans 3 or 4 years.

2. In-service Instructional Methods: The in-service instructional methods for the above training model an inquiry-based, student-centered classroom. For example, in IMP in-service sessions, teachers are seated in groups of four, like their students. The in-service presenters guide teachers through the units as they would their own students. Teachers work on selected problems in each unit. In addition to new math content, the training also focuses on student-centered instructional strategies and various forms of assessment, including portfolios, long-term problem-based essays, and group presentations.

An integral part of training is for teachers to actually teach all levels of a full-replacement curriculum. The in-services focus on practical issues of classroom implementation. Teacher manuals, videotapes and electronic networking supplement the in-services.

3. Promoting Teacher Leadership: Teacher leaders can be important agents of systemic change are within schools (Day, Goertz, & Floden, 1995). Teacher leaders are important because they: 1) act as agents of change within their building, 2) persuade their colleagues to take risks involved with change, 3) provide programmatic stability in the face of administrative turnover, 4) persuade parents of the need for change, and 5) become better classroom teachers. We promote teacher leadership by providing teachers with the opportunity to co-present in-service sessions with more experienced presenters, including the co-directors. Teachers leading in-services encourages the development of teacher-to-teacher networking and collegial support. Therefore, one goal of this project is to train more veteran teachers and mathematics department heads to lead or co-present in-service sessions. For example, over two dozen veteran Philadelphia IMP teachers have been involved in presenting or co-presenting IMP in-service sessions to new teachers in Philadelphia, New York and Boston. Experienced teacher leaders are utilized as much as possible to help schedule and conduct in-services for newer teachers.

4. Classroom Teacher Mentoring: As a follow-up to the in-service sessions, teachers are regularly visited in their classrooms to receive one-to-one mentoring. Secondary math teachers who were traditionally schooled must not only shift their thinking about the way students learn, but must also adopt different approaches to classroom management and student assessment. At the same time, many teachers must re-learn a large volume of mathematics content that is not usually taught in high school, such as probability, statistics, matrix algebra, and linear programming. Teachers also need to master the use of graphing calculators and must work through many unfamiliar, non-routine problems. New IMP teachers, for example, are regularly visited in their classroom, by a veteran IMP teacher-mentor or IMP director. During these visits, a mentor may team-teach part of a lesson or help lead classroom discussions. After each class, the mentor provides feedback to the math teacher.

5. Administrative Technical Assistance: The project directors provide technical assistance to principals, mathematics department heads, roster persons and other administrative personnel concerning:

1) Budgeting support for program implementation in their school;

2) Classroom materials, book, and calculator requisitions;

3) Teacher and student recruitment and classroom rostering issues;

4) Student transfer, absentee, retention and readiness issues;

5) Intra and inter-school and grade articulation issues;

6) Student test performance and program evaluation;

7) School-based implementation issues, such as ESL, special education inclusion, college admissions, NCAA, and AP Calculus and AP Statistic courses.

Program Logistics

1. Schedule: One half of the training takes place during the months of June, July and August, and occurs in 5-day blocks, 6 hours a day. (Lunch is an additional 45 minutes). Approximately 10 weeks is available for summer training. The academic-year training typically occurs on Saturdays or on other in-service days for 6 hours each day. Approximately twenty-two (22) Saturdays per academic year are available for training. (In-service sessions may run concurrently during the same summer week and on Saturdays.) All training dates are scheduled at the most convenient times for teachers. Teachers are also permitted to attend in-service training scheduled at other times with other participating school districts.

2. Location: Depending on the number of teachers per course, the training occurs either at La Salle University, or on-site in a school district, or another nearby school district. We strive for a class size of about 20 participants. A calendar of the dates, times and locations of summer and academic year training is distributed to all teachers by the end of the month of March preceding the in-services.

3. Classroom Mentoring: Teachers who have undergone training during the summer are provided with classroom mentoring during the following academic year. Teachers are visited an average of 8 times during the first year of their training; 4 times during the second year and 2 times during the third year. Each mentor has a set of teachers and schools as his/her responsibility. It is expected that each mentor will visit three different teachers per day, usually within the same school. Each visit entails a pre- and post-conference with the teacher. Each mentor submits a brief report of every visit. Periodically the mentors meet to discuss the progress of their teachers.

4. Administrative Technical Assistance: The co-directors and other project specialists will provide the technical assistance to schools. The average number of project days per year devoted to providing technical assistance for schools is approximately 8 director-days per school district, which includes meetings and telephone conferences. The number of consulting days diminishes each year until there is sufficient implementation expertise at each school site.

Criterion for GPSMP Participation

All participating GPSMP schools are selected based on three criteria:

1. Administrative Support: The administrative staff of a school district--superintendent, curriculum supervisors, school principals, roster chairs and mathematics department heads--have to be willing and prepared to commit school-based and district resources to support sustained multi-year teacher enhancement. In particular, they have to commit to a local cost share in the form of books, classroom materials, audiovisual equipment, graphics calculators and adequate classroom space, including desks for students to work in groups.

Principals must be visible supporters of change in their schools. This means teachers must have stable teaching assignments as they undergo extensive staff development over time. Teachers-in-training must also be provided the time during the school day to plan the use of new curriculum materials and discuss their classroom experiences with other teachers. For example, in Philadelphia high schools, IMP teachers-in-training have typically received a reduced course load or compensation for an extra preparation period while in training.

2**) School District Policy**: Perhaps the greatest single challenge to institutionalizing local systemic reform is maintaining the continuity of the reform process in the context of administrative personnel changes. For this reason, it is important that school districts adopt policies and practices, which will continue the change process with different administrators. The following policy indicators are used to select schools:

a) Adoption of standards-based curriculum and assessment frameworks;

b) Previous professional development on the NCTM *Standards;*

c) Willingness to align other professional development resources with this project (such as Title 1, Eisenhower, and Goals 2000 fund);

d) Adoption of administrative policies to support classroom reform, such as ensuring teachers have stable classroom and building assignments from one year to the next;

e) Provision for all students from diverse racial and economic backgrounds to have equal access to an inquiry-based, student-centered classroom;

f) Development of parent and community relations outreach and information program;

g) Alignment of new math teacher hiring criteria with the NCTM *Teaching Standards.*

3) Teacher Readiness: Many math teachers are simply not ready to undertake change. As of 1993, according to the National Science Board's *Science and Engineering Indicators* report (1996), 44% of high school math teachers and 72% of middle school math teachers were **not** "well aware" of the *NCTM Curriculum and Evaluation Standards*. An even greater percentage were **not** "well aware" of *NCTM's Professional Teaching Standards*. And, 44% of high school math teachers surveyed had 6 or less hours of in-service per year.

One major outcome of NSF's support for urban and statewide systemic change efforts has been to increase teachers' familiarity with the *NCTM Standards* and their implications for changes in their own classroom practice regarding curriculum, instruction and assessment. Nevertheless, teachers vary in respect to their own readiness to change. We look for schools with a sufficient critical mass of the school's teaching staff willing to participate in intensive, sustained professional development necessary to implement standards-based change.

Partnership Development Process

The process of forming a partnership between the GPSMP and a participating school district usually involves the following steps:

1) An initial invitation to participate or a request from the school district for an initial discussion,

2) Meetings between the project co-directors and key administrative personnel

3) Meetings with the mathematics supervisor or department head and teacher leaders,

4) A 2 to 3 hour presentation before the mathematics teaching staff,

5) Teacher visits to other schools,

6) Four days (24 hours) of in-service featuring two replacement units teachers can use,

7) A series of meetings to plan the types of professional development in-services and to work out logistical and funding details,

8) Presentation and/or approval from the school board,

9) A letter-of-commitment from the school superintendent,

10) Further meeting to plan in-services and classroom implementation.

School District Obligations: Each school district is responsible for:

1) Providing incentives for teachers to attend the in-services,

2) Providing extra time for lesson planning during the academic year,

3) Purchasing books, graphics calculators, overhead projectors, overhead graphics calculator, computer software, and classroom durable and consumable materials.

Costs: The typical *first* year per teacher costs of these items is:

1) Teacher in-service stipends (or graduate credit)	$1,000
2) Graphics calculators (1 classroom set = 35)	2,870
3) Books (2 classrooms sets)	2,520
4) Classroom materials	300
5) Audio Visual Equipment	700
	$7,390

Project Management

1. Project Directors: **Mr. Joseph Merlino, Dr. Edward Wolff**, and **Dr. Alice Jordan** are the directors and co-principal investigators of the project.

F. Joseph Merlino, M.A., Education, is the PI/PD for the Greater Philadelphia Secondary Mathematics Project and provides technical assistance to schools, supervisors all the mentors and presenters.

Edward Wolff, Ph.D., Mathematics, is a co-pi for the Greater Philadelphia Secondary Mathematics Project and has been a co-director of Philadelphia Regional IMP Center. He is also Chair of Mathematics and Computer Science Department at Arcadia University. Dr. Wolff provides training in IMP Level 4 training, Harvard Reform and Statistics in-services, mentors teachers and conducts statistical analyses of student outcomes.

Alice Jordan, Ed.D., Mathematics Education, has been a co-director of Philadelphia Regional IMP Center at La Salle University for 6 years while also being a Department Head at Strawberry Mansion High School where she taught IMP for three years. Now retired from high school teaching, Dr. Jordan constructs the in-service calendar and mentors teachers.

2. Additional Key Staff: Assisting the project directors are an experienced cadre of other in-service presenters and in-classroom teacher mentors familiar with IMP and Core-Plus and various NSF middle school curricula. Approximately 95 other teachers and university faculty are involved with providing in-services and mentoring to schoolteachers. A sample of these personnel are listed.

Barbara Stankus, an IMP 4 teacher on special assignment from Strath Haven High School in Wallingford/Swarthmore School District.

Anthony Campione is a recently retired IMP teacher from Furness High School. He has taught two levels of IMP and has co-presented numerous IMP workshops and has mentored dozens of new IMP teachers.

Richard Clancy is a recently retired IMP 3 teacher from Girls High school in Philadelphia. He has taught three levels of IMP, has co-presented numerous IMP workshops and has mentored dozens of new IMP teachers

3. Institutional Involvement: *La Salle University* is the fiscal agent for this project. Project partners include the *Pennsylvania Department of Education Mathematics Office* and the *New Standards Project in Education-University of Pennsylvania*. Fifteen schools from Delaware county, Montgomery county and Bucks county and the Center for a Greater Philadelphia have joined together to form the *Southeastern Pennsylvania Standards Consortium*. This consortium is based on the work of the *New Standards Project* at the University of Pittsburgh.

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Pending Research: 1999, 2000, 2001, 2002

- Strath Haven Studies: ERBs, SATs, PSATs, CORE-Plus Algebra Test, NAEP
- New Standard Reference Exam
- New York Math Regents Exam (New Year City Research Study)
- Philadelphia Community College Analysis
- New Jersey GEPA + HEPA state tests
- Pennsylvania System of Student Assessment (PSSA)