

## Professional Development Case Description

### *“The Game of Pig”*

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#### **1. Please provide a brief description of the instructional unit, including approximate number of hours of classroom instruction involved.**

The instructional unit is called *“The Game of Pig.”* It is the second unit of five in the first year of the *Interactive Mathematics Program*, a four-year high school curriculum. Pig is an introduction to the core concepts of probability and expected value as well as the supporting ideas of “randomness,” “equally likely,” “independent events,” “in the long run,” “theoretical versus experimental probability,” and “mean, median and mode.”

These concepts are contextualized in the unit problem, which involves determining the best strategy to win the most points in the *Game of Pig*. The game is simple. A person rolls a die for as many times as they wish adding up each roll of the die until they choose to stop or get a 1. If they voluntarily stop *before* rolling a 1, the person gets to keep all their cumulative points for that turn. But if they should roll a 1 at any time during their turn, the entire cumulative points for that turn of rolls is zero. The problem is to determine a decision rule for when to stop so as to maximize a person’s grand total points over many turns.

The concepts implied in solving *Pig* are represented in various ways using histograms, tree diagrams, area models or “rugs,” and sample spaces. Additional activities and games are used during the course of the unit. Gaming materials include dice, coins, playing cards, and spinner games. Several “Problem of the Week” or POW’s are included within the unit that require students to ponder how to solve a situational problem individually or in groups. Students individually submit a written explanation of their solution(s) to the POWs and the reasoning behind their answer. POWs are in addition to regular class work and homework assignments.

The *Game of Pig* is designed to be completed in approximately 30 school days assuming a 50-minute class period, a trained and experienced IMP teacher, and on-grade level students.

#### **2. What did you expect would be the challenges for teachers in teaching this unit?**

For teachers new to the *Interactive Mathematics Program* and student-centered pedagogy, the challenges are many. This unit can seem daunting if not overwhelming to a teacher. Listed below are the different kinds of challenges new IMP teachers must overcome to properly teach this unit.

**New Mathematics Content:** Most teachers have not taught probability or statistics. They usually last took the subject years ago while in college and have long since forgotten much of it. Certainly, the mathematics content was not presented in the manner of IMP. As a result, it is not unusual for new IMP teachers to struggle with the mathematics content at first, particularly near the end of the unit when the unit problem is finally solved.

A companion content problem for new teachers is to see how a particular concept is developed over time and related to other concepts within the unit. Teachers do not as yet have a “cognitive map” of either the flow of the unit or its content. As a result, teachers tend to teach each day’s lesson in isolation without providing students opportunities for insight into the interconnections of activities and topics. It is like pounding each note when one is first learning to play the piano rather than making music, or like self-consciously taking awkward steps rather than dancing fluidly.

**Different Classroom Resources:** Implementing IMP requires a significant number of physical changes within the classroom. The classroom space needs to be large enough to accommodate groups of desks or tables while still allowing ease of movement by the teacher between and among them to reach all students. The classroom should have either tables or flat desks. The teacher needs a convenient storage place for graphing calculators, manipulatives, chart paper, student portfolios and a variety of durable instructional aides such as Geoboards. A white or blackboard is needed. A classroom set of graphing calculators, an overhead graphing LCD panel, overhead projector, screen and an electrical outlet are required. The room should also have sufficient wall space so that student work can be easily posted and removed. Teachers must effectively and efficiently learn to use graphing calculators, manipulatives, chart paper, games, and the overhead projector within the *Pig* unit.

*The Game of Pig* utilizes many hands-on activities within each unit to initially provide students an intuitive sense of the problem and/or concept. Teachers, however, tend to become preoccupied with the logistics and proper mechanics of managing an activity. As a result, teachers often end up teaching the activity as if that was the lesson, rather than using the activity as a vehicle to develop the mathematical concept. Thus, a challenge for teachers teaching *Pig* is be ever-mindful of the mathematical purpose of the unit and not to allow themselves to become mired in simply doing a series of *Pig* activities, day after day, as if that was the point of the unit.

**Teaching a Thematic Unit:** The *Pig* unit is the first thematic unit new IMP teachers encounter. It is the first unit where there is a large unit problem that requires four to five weeks to solve. New IMP teachers have difficulty with pacing, prioritizing the various activities, homework, and daily lessons. They are unsure of how much to emphasize certain sections of the unit rather than others, what remedial instruction students may need, when to move ahead with the lesson, how to plant “seeds,” and when student’s conceptual understanding should ripen in the unit.

New IMP teachers have difficulty linking the individual parts of a day’s lesson into a coherent whole, and then linking the many daily lessons into a larger unit. Moreover, they don’t see the relationship of the *Pig* unit to other IMP units and how student understanding is

progressively scaffolded in succeeding units. Teachers often become so preoccupied with simply getting through the day's lesson that they present it devoid of its original motivating purpose. As a result, lessons can degrade into mere "exercises" and miss the overarching educational goal.

**Organizing a Lesson for Conceptual Understanding:** Perhaps the most difficult challenge for new IMP teachers is to teach a unit like *Pig* for student conceptual understanding not mere computational proficiency. In teaching for conceptual understanding, teachers have a great deal of difficulty not talking and not being the absolute center of attention 100% of the time. In *Pig*, the unit problem takes center stage. Conceptual development occurs in a seven-step process of:

1. Posing a problem or question to the student,
2. Eliciting individual intuitions and prior student knowledge, preferably in writing,
3. Having them experience a physical simulation or model of the problems,
4. Encouraging students to individually reflect upon and write about their experience,
5. Directing students to exchange their reflections and ideas, and/or doing another activity in small groups,
6. Sharing results as a whole class and discussing similar and conflicting analyses
7. Repeating steps 1-6 with a more complex problem or further extension of it.

Most mathematics teachers will say their goal is to produce "independent thinkers." At the same time, teachers find it a personal challenge to let go and allow students to think and be independent, even within a unit specifically designed with such reflection and independence.

**Encouraging Student Presentations:** A closely related challenge for teachers is to always look for ways to have the students do the talking and presenting. Judiciously allowing students to use the overhead projector, write on the board, address the whole class, read a problem out loud, and demonstrate how they solved a problem is very difficult for teachers to actually do in their classroom, despite their intellectual agreement about the wisdom of doing so. Using students to teach the lesson, a practice I call "indirect teaching," can be a very powerful way to develop conceptual understanding as well as to motivate students. People learn most when they teach. They get energized when they get a chance to talk and participate in the lesson. The *Pig* unit provides teachers abundant opportunity for such indirect teaching. But it is very difficult for teachers to let go of the chalk and center stage and hand it to a student--even for a moment.

**Making the Most of Student Grouping:** The *Pig* unit is supposed to be taught using small groups of four students each, arranged around a table or group of clustered desks. Many teachers, however, simply rearrange their traditional rows and columns of desk into a group of desks as the "default setting." This is their idea of "group work." Some teachers either rarely place their students in groups or feel they must do so all the time. Thus, a challenge for teachers is to understand the purpose of student learning groups. They are meant to promote individual conceptual development and mathematical communication skills. The groups allow students to discuss their ideas while affording the teacher an opportunity for individualized, small group instruction. Like hands-on activities, student groups are not ends in themselves.

**Questioning Techniques:** As the TIMSS video study shows, teachers usually ask three types of questions: 1) they ask the whole class rhetorical questions they then proceed to answer, 2) they ask for answers to assigned problems, or 3) they ask for definitions of terms. Teachers are

used to asking students *how* to do a procedure, not why. They don't ask the meaning of the topics they teach (although students often ask that question) or how it connects to other topics. They don't ask them to explain their reasoning or how they solved a problem because the problems usually presented in class are computational in nature. Teachers are used to explaining and telling, not listening and questioning students. In the *Pig* unit, there are many opportunities to pose counter-intuitive questions to elicit students' reasoning processes.

One of the best questioning methods is for teachers to literally sit down at a group table and do a Q & A session with the students. The teacher, who should probe further based on student responses, does the questioning. For example, in *Pig*, students are presented with the definition of probability as the ratio of favored outcome to all possible outcomes. They are assigned counting activities using coins and dice, which illustrate that definition. Questions can then be asked such as "when are you completely certain something will happen, what is the ratio? Can someone be 200% certain something will happen? Why or why not?"

A related challenge for teachers is to pose questions that are not immediately answerable but are left "pregnant." The whole *Game of Pig* unit is essentially a 30-day, open-ended question. Organizing instruction around complex, open-ended questions is difficult for teachers. They are accustomed to posing a problem and then immediately jumping in to show students how to solve it. Guiding students through a directed discovery approach requires new pedagogical skills, a deeper grasp of the course's mathematical content, a heightened sensitivity to the learner's state of mind, and a great deal more patience on the part of teachers.

**Assessing Student Understanding versus Grading Answers.** The answer to the unit question in *Pig* is "20." Using a directed instruction format, in a single period a traditional teacher could pose the *Pig* unit problem to students, show them an algorithm for solving the problem and arrive at the answer. The teacher would then test the students using a slightly different problem and move on to another problem rather than spending thirty days on *Pig* as IMP does. IMP does not supply teachers with an answer key. This is a common complaint among new IMP teachers who moan, "Why can't they just give us an answer key?"

Indeed, if genuine student understanding of *Pig*'s core and ancillary concepts of probability could be achieved in a single day, this traditional approach would be the preferred strategy. But as decades of mathematical educational research show, students who are simply taught algorithms to solve contrived problems acquire little in the way of mathematical insight and knowledge. The purpose in teaching the *Pig* unit is not for students to learn how to play the *Game of Pig*. *Pig* itself is irrelevant; it is merely a vehicle to achieve other more important mathematical and cognitive learning goals. Thus, another challenge for teachers teaching *Pig* is to shift from a mode of grading answers to assessing student understanding through the use of rubrics, student portfolios and written explanations of their problems solving strategies.

**Acquiring a Different Learning Theory Paradigm.** Teachers' ideas of how students learn are conditioned by our high technology, manufacturing, and building culture. Only 3% of people still live on the farms. As a result, our learning theories tend to be laced with mechanistic and building metaphors, such as "laying a foundation" and "skill building" as if teachers were laying bricks. The masonry/manufacturing model of teaching content is deeply ingrained in teachers. Many teachers tend to behave like warehouse deliver people, picking up facts from their

“storehouse of knowledge” and delivering “packages” to students.

Successfully teaching a unit like *Pig* for understanding, however, requires that teachers acquire a different learning paradigm, one that is organic and biologically based. It is a paradigm more akin to the rhythms and behaviors of farming or horticulture. In manufacturing, the manufacturer does all the work, building an inanimate product, step-by-step. But in making wine, the vinedresser cultivates growth with patience and care. Like cultivating any living thing, the act of understanding is a biological process, not a mechanical one. The mind is a living thing. This distinction is a challenge for teachers to appreciate and reflect in their practice, but it is essential if teachers are to optimize their students’ understanding of important mathematics.

**Achieving “Flow”:** Psychologist Mihaly Csikszentmihalyi in his 1990 book, *“Flow”* describes optimal experience as “the state in which people are so involved in an activity that nothing else seems to matter; the experience itself is so enjoyable that people will do it even at great cost for the sheer sake of doing it.”<sup>1</sup>

The goal of our professional development is to help teachers achieve “flow” with their students. Unless students have had a reform curriculum in at least middle school, most 9<sup>th</sup> grade students themselves require an adjustment to a unit like the *Game of Pig*. For this reason, new IMP teachers and even experienced ones who have only taught 9<sup>th</sup> graders, don’t get an opportunity to be “in the flow” with the IMP curriculum until the 2<sup>nd</sup> or 3<sup>rd</sup> year of the program. It is at this point when they are amazed at how absorbed students can become in the material and derive intrinsic enjoyment from learning mathematics.

**Aligning Instruction with High-Staking Testing.** When high stakes tests are used for accountability purposes, it is necessary to make explicit connections between the assessments used in the *Pig* unit with the particular style of questions used by the state test and the mathematics content of *Pig*, and vice versa. This demand places an additional, but unavoidable, extra burden on the first year IMP teacher.

### **3. Therefore, what were the challenges in providing professional development for this specific instructional unit?**

The PD challenges are both general and particular. They are general as regards the whole IMP program, and particular as regards the specific instructional unit e.g., the *Game of Pig*. In short, there is too much to learn and experience to teach this unit well for the space of two in-service training days. In a sense, the entire IMP training program also is training for each IMP unit. For example, being trained in the Chi-square IMP 2 unit called *Is Their Really a Difference* helps teacher understand the significance of the *Pig* unit.

### **4. How was the professional development structured to meet these challenges?**

The professional development is actually a package of training components rather than just

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<sup>1</sup> Mihaly Csikszentmihalyi, *Flow: The Psychology of Optimal Experience*. Harper Perennial, 1990. page 4

classroom type of in-service. These components consist of the following:

- Each IMP teacher receives twelve hours of in-service in each IMP unit, roughly two full days per unit, taught by a fellow teacher who is an experienced IMP teacher.
- Part of the Pig unit training is also to be trained in all the other IMP units to deepen teachers' content knowledge of probability as well as to understand how the IMP curriculum is structured.
- Each teacher can enroll in a five-day, IMP-based, graphics calculator course.
- Each teacher is encouraged to actually teach the unit they have been being trained in, at least for one semester.
- Each teacher is provided a mentor who visits their classroom and conducts before and after conferences with the teacher via e-mail and in person. Teachers are mentored 20 hours the first year.
- Each teacher and mathematics department is encouraged to meet each week to discuss common concerns, insight and experiences in the program. On-going collegial dialog involving focused reflects on practice is an essential PD component.
- Each mathematics department is encouraged to arrange peer teacher visitation with the school.
- Each mathematics department is encouraged to allocate resources and time for teachers to attend IMP-related professional conferences, such as NCTM meetings, regional IMP users conferences, and National IMP Teacher Leadership conferences.

**a. What did a typical teacher experience in professional development prior to teaching this unit for the first time?**

Teachers will have gone through at least 2 days of IMP in-service in the previous IMP unit, "*Patterns*." In the year previous to IMP implementation, they may have received one day of preparation training, 2 days of training in an IMP replacement unit such a "*Bakers' Choice*," and 2 days training in a CORE-Plus replacement unit that our LSC provided. Teachers may have received prior school-sponsored training in the NCTM, state mathematics, or NCEE New Project standards, 1-3 days of graphics calculator training, and/or training in state open-ended assessments and rubrics.

**b. How was a typical teacher supported as s/he taught the instructional unit for the first time?**

Teachers are supported usually through in-classroom mentoring by an experienced IMP teacher from our LSC project and special school departmental meetings, either during or after school.

**c. What did a typical teacher experience in professional development about this unit after teaching this unit for the first time?**

They received training *before* they taught this unit.

**d. How many hours of total professional development did the activities in a. – c**

**represent?**

It can vary between 24 hours and 54 –60 hours.

**5. Based on PI/project staff observations of a minimum of 3 teachers, in what ways does the instruction in this unit appear to be consistent with the project's vision and in what ways does it is it not?**

Three ninth grade teachers were observed teaching lessons from the Game of Pig Unit; two from suburban schools and the third in an urban setting. All three teachers were new to the IMP program this year. The first teacher, from one of the suburban schools, began the lesson by working on a homework problem on expected value. In the problem, the students imagine that they have two pockets and that each pocket contains a penny, a nickel, and a dime. They reach in and remove one coin from each pocket and determine the possible amounts they could get from the total of the two coins. The first half of the lesson was devoted to having the students present their solutions to the homework problem. The second half of the lesson involved the teacher instructing the students on how to use the graphing calculator and how to do a frequency bar graph on the graphing calculator. While there is a mathematical segue between the homework problem and frequency bar graphs on the graphing calculator, the teacher made no attempt to tie the two together. The teacher went over questions on the homework, but did not probe students further with extensions to determine if they understood the concept nor did she tie the problem to previous activities. The teacher was directing the students toward the correct answer to the homework questions in a lockstep manner. For the students, there was no apparent purpose to why she was teaching them how to use the graphing calculator to make frequency bar graphs.

The second lesson observed was also a teacher in a suburban school. The students, who were lower in ability level than the first class observed, sat in rows facing the teacher. The lesson was also from the Game of Pig but was about a different homework problem on expected value. The teacher had written the questions on the board and students came up to the board and filled in the answers. In going over the questions, the teacher played the major role in this lesson, talking most of the time while trying to get students to answer his questions using the "fill in the blank" format that he had set up on the board. The questioning was task-oriented in arriving at a product or a result, but lacked purpose aimed at developing students' conceptual understanding.

The final lesson observed was in an urban school. In this lesson students were working on expected value by playing the spinner game. In this game, "Al" and "Betty" were playing a game with a spinner. Al has 25% of the board and wins \$4 from Betty when the spinner stops on his portion of the board. Betty has 75% of the board and wins \$1 from Al when the spinner stops on her portion of the board. The students are trying to determine who is more likely to be the winner of this game, and if Al and Betty played 100 times who would be the expected winner. The students worked on the problem with the spinners for the entire period and the teacher did not provide any type of sense-making during the lesson. As in the previous lessons, the aim was to get the answer to the problem of Al and Betty, not to get them to understand that probabilities can have weighted values to them that affect your decisions. In addition, there was no tie between this activity and related activities they had previously experienced. In hindsight this gap was

foreshadowed by the instructional purpose written on the board at the beginning of the day: learn how to play the spinners game.

In all three lessons observed, teachers did not demonstrate that they understood the content or how the concepts in the lessons they were teaching fit into the concepts in the unit. They tended to zero in on the minutiae of a particular lesson and apparently did not recognize how the lessons fit into the bigger picture of the unit.

**6. Assuming that you devoted the same number of professional development hours to this unit again, how would you structure the professional development and why?**

The issues demonstrated in the observations are difficult to address within the format of the professional development program. Teachers need to change their perceptions of themselves as teachers, their own ideas of learning, and their understanding of how students learn mathematics. During the first year, a new IMP teacher's classroom practice is usually not very consistent with the project's vision. Teachers typically dominate the classroom and talk too much. They don't often allow students to present. They don't manage groups well. They don't understand well the mathematical purpose of the unit and don't have a sense of students' prior knowledge. They are unskilled and unartful about leading students' discovery. They grade too many papers and spend too much time on each one. They are very impatient with themselves and with the students. Some don't distribute or use the graphing calculators or use the overhead projector.

Most teachers do try hard and make a good faith effort to implement the curriculum as intended. Over time, by the end of the first year, many teachers make great strides in their content knowledge and pedagogy. But it can take five years or more for teachers to become truly comfortable and confident with the whole IMP program and skilled at managing a student centered classroom.

A further advance in our professional development package would be to infuse multi-media technology into the training, mentoring and after-school meetings whereby teachers could conveniently access visual vignettes illustrating different aspects of a student-centered lesson that are specific to a particular curricular unit, such as *Pig*. Such multi-media products might take the form of a DVD with video and hyper-linked textual references, including examples of student work, assessments and experienced teacher commentaries.