STANFORD UNIVERSITY
MATHEMATICS TEACHING AND LEARNING
STUDY:

Initial Report – A Comparison of IMP1 and Algebra 1 at Greendale School.

Principal Investigator: Dr Jo Boaler.

Researchers: Karin Brodie, Hermione Gifford, Victoria Hand, Kristen Pilner, Emily Shahan, Melissa Sommerfeld, Megan Staples, Judy Strauss, Sean Whalen, Toby White, Rachel Chou.

This material is based upon work supported by the National Science Foundation under Grant No. 9985146. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science

Foundation.

1. Introduction and Overview.

Abstract.

The following report gives initial results from the first year of a study of mathematics teaching and learning, conducted by researchers from Stanford University. The study is designed to track and compare students' progress through different mathematics curricular and teaching approaches. In the school discussed in this report, two approaches: a traditional algebra and geometry sequence, and an alternative problem-based integrated mathematics approach, are offered concurrently. This school is a particularly interesting place in which to study mathematics approaches, as the students were taught mathematics in different ways, but they come from the same school population and have had similar mathematics teaching experiences in the past. Results from the first year of this study indicate that there were no significant differences in tested achievement of students taught through the different curricular approaches. Results also show that students in the different programs are demonstrating important differences in attitude and interest towards mathematics. These differences are particularly important because they are likely to affect the students' subsequent learning of mathematics.

Introduction.

The following report gives initial results from the first year of a study of mathematics teaching and learning, conducted by researchers from Stanford University. The study is based in three high schools, including Greendale[1] High School. Greendale offers students (and parents) a choice between the Interactive Mathematics Program (IMP) and the traditional algebra-geometry sequence. In the IMP program students work on applied problems that include content from algebra, geometry, statistics, and probability in each year of the course sequence. In the 'traditional' sequence of courses students study algebra for a year, then geometry, then advanced algebra etc. Students in the traditional sequence start with algebra 1, which is taught using the textbook: *Algebra: Structure and Method* (Brown, Richard G., Mary P. Dolciani, Robert H. Sorgenfrey, and William L. Cole, 2000, Evanston, IL: Houghton Mifflin , McDougal Littell Inc.). In algebra 1 teachers demonstrate methods that students practice by working through exercises, generally independently. Students in IMP start with *IMP 1*, the first text of the 4-year IMP series (Fendel, Dan, Diane Resek, Lynne Alper, and Sherry Fraser, 1997, Emeryville, CA: Key Curriculum Press). In IMP 1 teachers also demonstrate methods, but students are required to use these in solving broader problems, and students complete much of their work in groups. During the 2000-2001 school year a team of researchers at Stanford University monitored the students who were taking either IMP 1 or algebra 1 at Greendale (algebra 1, n = 80, IMP 1, n = 46). The different curricular approaches offered by the school presented an important opportunity for Stanford researchers to study the impact of these different approaches, as the students had a common mathematics background from their middle school education, but were taught mathematics in different ways from ninth grade onwards. This is an interim report that sets out some of the main results from the first year of the study[1]. In the following text we will describe some important results relating both to mathematics achievement and to student interest and engagement.

---

[1] Additional results from year 1 data will be available at a later date.

At the beginning of algebra 1 and IMP 1 we constructed and administered a test of the mathematics that students should know at that time, drawing from the middle school mathematics curriculum. The different questions assessed ratio, percentage, fractions and algebra. There were no significant differences in the attainment of students beginning algebra 1 and IMP 1 on that test (algebra 1: mean = 51%, standard deviation = 8.37, n = 90; IMP 1: mean = 52%, standard deviation = 8.18, n = 47; t = 0.44, *n.s.*). This result indicated that the students starting the different approaches had reached similar levels of mathematics attainment. Over the following year researchers then analyzed the mathematics teaching and learning in the two approaches carefully, examining the different factors that impacted the students' developing understanding. Research methods included observations of approximately 100 hours of mathematics lessons in the different approaches, most of which were videotaped for close analysis and for multiple coding. Every math class was observed at least once and three focus classes were observed and videotaped on multiple occasions. We administered questionnaires asking the students about their mathematical experiences and their resulting beliefs and attitudes towards mathematics. We also interviewed students from each of the focus classes. Both successful and unsuccessful students in each class were selected for interview in order that we may investigate the factors impacting success and failure *within* each approach. In addition to observations, questionnaires, and interviews, which enabled us to gain information on the students' teaching and learning experiences, we monitored the students' knowledge and understanding. At the end of the year we gave students a test that assessed some of the algebraic concepts and methods to which students had been introduced over the year. In writing a test that would carefully and fairly assess students who had learned in different ways, we needed to make sure that all students had encountered the mathematics assessed. This meant that the test only assessed algebra, even though the IMP students had also learned about geometry, statistics, and probability. We also made sure that the test was consistent with the *types* of questions students were used to in the different approaches, so some of the questions were typical of the traditional algebra curriculum (e.g. Solve the equation: 5x-3=101), whereas others were more typical of the IMP curriculum (e.g. An amusement park charges $10 for the first 3 rides and $2 for each additional ride, another charges $15 for 3 rides and $1 for each additional ride. Which park is better value?).

2. Results & Analyses.

Algebra Achievement.

When the students started their different approaches there were no significant differences between the achievement of students choosing to take algebra 1 or IMP 1. In the algebra test administered to all the students one year later, there were again no significant differences in achievement (algebra 1: mean = 46%,

standard deviation = 11.29; IMP 1: mean = 40%, standard deviation = 8.90; t = 1.62, *n.s.*)[2]. The similarity in achievement of the students was a particularly positive result for the IMP students, who spent the year learning about algebra, geometry, probability and data analysis, but performed as well on a test of algebra as students who had learned algebra all year. The similarity in performance was achieved through students achieving similarly on most questions, rather than students succeeding only on those questions that were most similar to their own approach. For example, in the question cited above (Solve the equation: 5x-3=101), the mean scores for Algebra 1 and IMP 1 were 54% and 57%, respectively.

The fact that students achieved at approximately the same levels after one year of working on different curricula was an interesting result, particularly given the vast amount of time and energy that has been spent arguing about different curricular approaches in the United States. Both approaches of algebra 1 and IMP 1 were taught well at this school (all classes scored in the top half of our total sample of classes) and both approaches gave students access to the knowledge they needed to succeed on tests of mathematics. Our research also showed, not surprisingly, that the particular decisions the teacher made in their enactment of the curriculum were extremely important in the creation of positive learning environments. Teachers of algebra 1 or IMP 1, who created serious work environments in which students felt encouraged to learn, who supported students by giving clear explanations and asking helpful questions, and who showed a clear commitment to the needs of different students, enabled important learning to be achieved, whichever curriculum they were using.

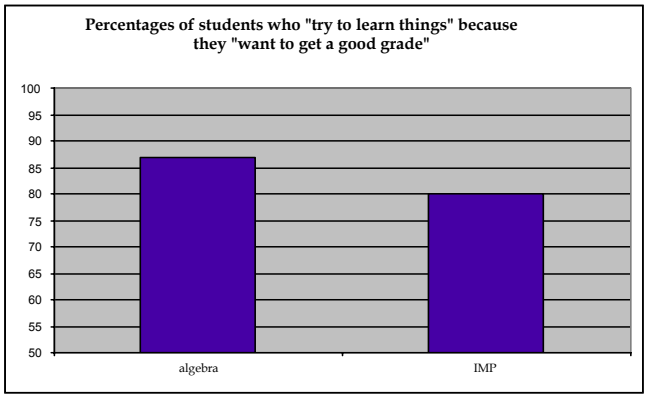
While the differences between the two curricula approaches did not—alone—appear to impact test performance, they did have an impact upon the mathematical beliefs and attitudes that students developed, as will be described below.

Students' Attitudes Towards and Interest in Mathematics.

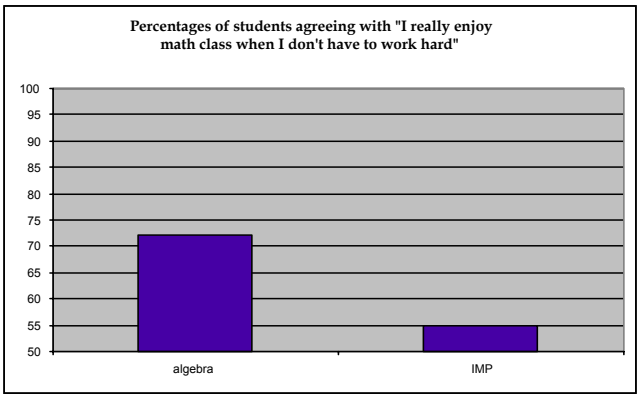
Despite the similarity in achievement of students taught in different approaches, our research indicates that students are developing mathematical beliefs and attitudes that are quite different from each other. Students' attitudes towards mathematics are extremely important – they play a large part in determining students' motivation towards lessons and their subsequent achievement, as well as influencing the number of courses they take in mathematics and their ultimate relationships with a subject that they will continue to use throughout their lives (Eccles, 1993; Eccles 1994). One focus of our questionnaires was the motivation that students were developing towards mathematics. Students may be motivated to work hard for a number of reasons – for example, they may generally be motivated to achieve high grades or to gain confirmation of their ability (Dweck, 1986) or to do as well or better than their peers. All forms of motivation are

[2] The average scores for all students appear low, but the test included a number of difficult items and drew from work from across the previous year.

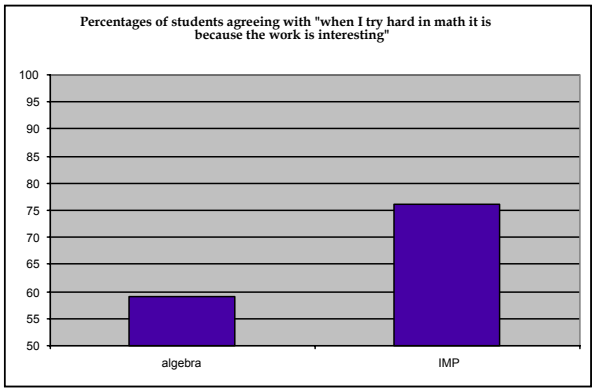
important, but we know from research that a particularly productive form of motivation is that which is based upon *intrinsic interest* (Ryan, 1992) in the subject being learned. When students develop an interest in and enjoyment of mathematics and strive to learn something for the sake of learning rather than only to achieve a particular grade, they are better placed to develop positive and long-term approaches to learning (Dweck, 1986; Ames, 1990). In our questionnaires we included a number of items that probed students' motivation, looking specifically for an intrinsic interest in mathematics. The results indicated that the students taught in different ways are developing different motivations. For instance, the responses of the algebra 1 students indicate a stronger orientation toward grades, while the IMP students display higher levels of intrinsic interest. Specifically, more of the algebra 1 students said they would try to learn new mathematics because they wanted to "get a good grade":



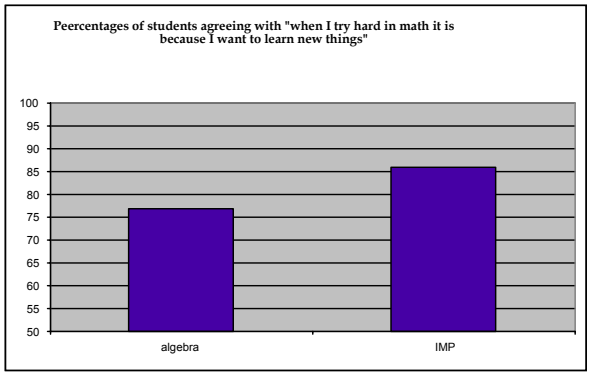
And more of the algebra 1 students agreed with the statement: "I really enjoy math class when I don't have to work hard":



Whereas more of the IMP students said that they tried hard in mathematics "because the work is interesting":



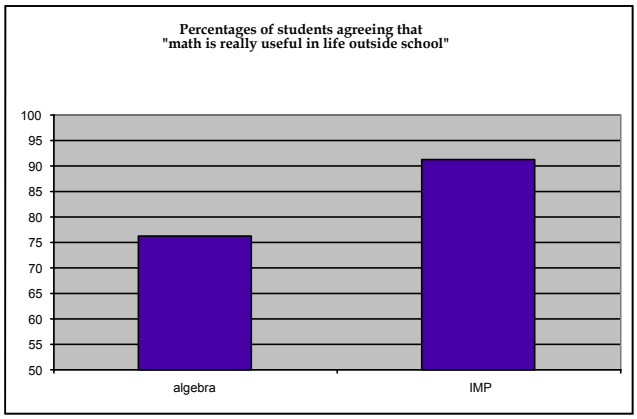
And more of the IMP students said that they tried hard in mathematics because they wanted "to learn new things":



It is important to note that there are large degrees of overlap between students taught with the different curricula, but the differences that are emerging are important and worthy of careful consideration. One might reasonably question whether the students entered the different curriculum approaches with different attitudes towards mathematics, but this seems unlikely. One of the schools in our study offers no choice in curricula and all students learn through methods similar to those in IMP classrooms. At that school the students responded in the same way as the IMP students on all of the questions shown above, suggesting that the students' responses were related to the teaching methods they experienced in class, rather than any predispositions they held prior to the start of their different approaches. The direction of the difference, with IMP students appearing to have more intrinsic interest, is in some ways unsurprising as "reform" curricula such as IMP were designed, in part, to increase students' interest in mathematics. The inclusion of problems that are related to life and that students are required to solve means that students do more than reproduce abstract methods and procedures in mathematics class. Students also engage in problem solving,

hold discussions about the mathematical directions they should follow, and see the applications of mathematics, increasing the potential for engagement and interest.

Another important difference between the two groups of students occurred in questions asking the students about the usefulness of mathematics. The IMP students were much more positive about the usefulness of mathematics for their lives:



These differences suggest that the contrasting curricula are having an effect upon students' interest, and it will be important to monitor these differences as students continue with their different approaches.

Interviews with students in the different approaches showed that most students were satisfied with their teaching experiences and students talked positively about mathematics, but the interviews also revealed that students in different approaches were developing strong differences in their beliefs about the nature of mathematics and—importantly—their approaches to learning. The students who learned through traditional algebra generally reported that the teacher presented standard methods and rules, and the students' job was to remember them and use them in different questions. In contrast, the students who learned through IMP reported that they were expected to think about and explore the mathematics they encountered, to discuss what the different methods meant, and to contribute their own mathematical ideas.

In the traditional algebra classes the students reported that their role in the mathematics classroom was to *receive* the knowledge the teachers presented, and to reproduce it. They did not stress thinking as an important aspect of mathematics. Instead they talked about the need to follow the methods the teacher showed them—exactly—and to remember them. For example:

> "He teaches you exactly how to do it. He'll show us tricks that he knows. And he'll always give us the easiest way (…) You don't have to have big thoughts. There's simple little steps that you just follow"

> "Make it, like a habit to do it. Just don't even think about it. Just like, okay, now what are the next steps?" (algebra 1 students)

The students appreciated the clarity of the mathematical steps and 'tricks' they were shown, and they strove to remember them, but the students we interviewed did not perceive their role in the mathematics classroom to be that of thinking deeply about the mathematics they met. The students' views about the nature of mathematics after one year of algebra were fairly one-dimensional—you get it or you don't. They did not appear to be developing an interest in mathematics as a subject—an appreciation of the different ideas in mathematics, and ways of solving problems, or the relationships within mathematics and between mathematics and their lives. The students talked, instead, about being motivated to attain a good grade, for example:

> "I've never used algebra outside of math class"
> "Right now the only thing I can really think of algebra is that it's ten credits. Right now"
> "It's like finishing the semester, getting the points, getting it done." (algebra 1 students)

The algebra students were very appreciative of their teachers and the ways they made the subject accessible; their main complaint was that they could not see uses for the subject they were learning. Students told us that they would have liked the opportunity to see the ways mathematics may be used and applied:

> "I just want to know when we'll use that. That's the one thing I wish they would teach, is when you'll use it. Like they need to give us examples of it, otherwise, no one's going to listen" (algebra 1 student)

The students taking IMP expressed similar degrees of satisfaction with their mathematics experiences, and appreciation of their teachers but they talked in very different ways about the nature of mathematics. The students talked about the different *dimensions* of mathematics that they appreciated. Their interest and enjoyment often stemmed from the applications of mathematics that they could see and use. For example:

> "The one we're doing right now is good because it's based on something that really happens. It's based on how the pendulum swings and it's really connected to that. And when we learn something in class he [the teacher] really goes into depth and we look at it from different angles and we know how things work and we know how to use them. (…) It's really making sure that we know how to connect the math to what's happening in life." (IMP 1 student)

The students felt that they were able to adapt and apply the mathematical methods they had met in ways that had not previously been possible for them:

> "I think it'll be useful on, like, important tests - SATs and stuff. Because in the past when we just learned 'use this formula for this problem', if on a standardized test we saw a problem that was close to it, but there was just one little thing that was different then you couldn't figure it out, because it

was so set. But now I know how a problem works and stuff. And if there's something different about a problem I can usually use it." (IMP 1 student)

The flexibility in approach that the students reported, and the encouragement they received to think deeply about mathematics probably helped them achieve similar levels on the algebra test, despite their relative lack of experience with algebra, compared with those who had taken algebra 1.

3. Summary.

The early stages of our research suggest that there is a large degree of overlap between the test achievement of students in the different curricula approaches, but also that the different programs are producing differences in attitude and interest towards mathematics and approaches to learning and using mathematics. These differences are important and they are likely to affect the students' subsequent learning of mathematics. We will continue to monitor these as we follow students through high school.

This study gave us the opportunity to study algebra 1 and IMP 1 taught well, by good teachers. If these broad curricular approaches are not taught well then completely different results are likely to emerge. These results suggest that the different curricular approaches, well-taught, are likely to produce similar achievement on standard tests. However, the curricula also seem to promote some important differences in students' approaches to mathematics and attitudes towards and beliefs about mathematics.

The similarities in achievement of students taught through different curricula seems important to reflect upon as many of the teachers in our study have been severely negatively affected by local events in which decisions they had made about curricula for their students had been undermined. As they fought to defend their choices—and their professionalism—they were able to devote less and less time to their own learning about effective teaching methods. One clear result of this research study is that it is the *particular* methods that individual teachers use in their classrooms, rather than general curricular and teaching approaches, that make the difference between more and less effective learning. This is not to say that curriculum is not important, but that it is the teachers' implementation of the curriculum that should be the focus of attention. It is imperative that greater support be given to the work of teachers in classrooms and to the development of particularly effective teaching practices, whichever curricular approach they are using.

If you would like to be placed on our email list to receive more information on this study as it emerges, email joboaler@stanford.edu

References:

Eccles, J.S. (1994). Understanding women's educational and occupational choices. *Psychology of Women Quarterly, 18*, 585-609.

Eccles, J. (1993). School and family effects on the ontogeny of children's interests, self-perceptions, and activity choices. In J. Jacobs (Ed.), *Developmental Perspectives on motivation, Vol. 40 of the Nebraska Symposium on Motivation* (pp. 145-208). Lincoln: University of Nebraska Press.

Dweck, C. (1986). Motivational processes affecting learning. *American Psychologist, 41*, 1040-1048.

Ryan, R.M. (1992). Agency and organization: Intrinsic motivation, autonomy, and the self in psychological development. In J.E. Jacobs (ed.), *Nebraska Symposium on Motivation: Developmental Perspectives on Motivation* (pp. 1-56). Lincoln, NE: University of Nebraska Press.

Ames, C. (1990). Motivation: What teachers need to know. *Teachers College Record, 91*, 409-421.